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EFFECTIVENESS OF AIR-DROPPED ANTI-SUBMARINE TORPEDOES

RICHARD C. HANDFORD

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Richard C. Handford

EFFECTIVENESS OF AIR-DROPPED ANTI-SUBMARINE TORPEDOES

by

Richard C. Handford
Lieutenant Commander, United States Navy

Submitted in partial fulfillment of
the requirements for the degree of

MASTER OF SCIENCE

United States Naval Postgraduate School
Monterey, California

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ABSTRACT

The decision problem of the allocation of available weight and space in an air-dropped anti-submarine torpedo to fuel, explosive, and to attack speed capability above some preset minimum is studied for the case in which alternatives under consideration do not differ with respect to their effect on weapon reliability and to their effect on enemy countermeasures capability. Analytical models are developed which relate (1) probability of acquisition to endurance for the case of circling search at constant depth, (2) probability of hit given acquisition to endurance and attack speed, (3) probability of hit on a close-in attack run to attack speed, and (4) probability of kill damage given a hit to amount of explosive used.

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TABLE OF SYMBOLS AND ABBREVIATIONS

A_j = area in the j th plane which is scanned during one ping cycle (yd^2); $j = J', \cdots, J''$.

$A_{jk}(\beta)$ = echo excess in plane j at a range of 100k yards which is required to acquire a target at an azimuth angle (β) to the direction of the transmitted ping (db); $J = -J, \cdots, J$; $k = 1, \cdots, K_j$

AMV_{du} = effective volume in level L_d above weapon search depth but below the critical depth missed inside the insonified volume on ping zero on the u th search turn (yd^3); $d = \overline{D} + 1, \cdots, \overline{D}'$; $u = 1, \cdots, U$

ASV_{du} = effective volume in level L_d above weapon search depth scanned on ping zero on the u th search turn (yd^3); $d = 1, \cdots, \overline{D}'$; $u = 2, \cdots, U$

B_{jk} = echo excess in plane j at a range of 100k yards available for acquisition on the azimuth reference axis. (db) $j = -J, \cdots, J$; $k = 1, \cdots, K'_j$

BMV_{du} = effective volume in level L_d below weapon search depth but above the critical depth inside the insonified volume on ping zero on the u th search turn (yd^3) $d = \underline{D} + 1, \cdots, \underline{D}'$; $u = 1, \cdots, U$

BSV_{du} = effective volume in level L_d below weapon search depth scanned on ping zero on the u th search turn (yd^3); $d = 1, \cdots, \underline{D}'$; $u = 2, \cdots, U$

BW = length of the beam dimension of the target (yd)

b_1 = normalized pattern function in the vertical plane for transmitting pattern (a function of vertical off-axis angle)

b_2 = normalized pattern function in the vertical plane for receiving pattern (a function of vertical off-axis angle)

b_3 = normalized pattern function in the azimuth plane for transmitting pattern (a function of azimuth off-axis angle)

b_4 = normalized pattern function in the azimuth plane for receiving pattern (a function of azimuth off-axis angle)

CDA = critical depth above search depth above which a target trapped inside the insonified volume may escape and below which there is no escape (ft)

CDB = critical depth below search depth below which a target trapped inside the insonified volume may escape and above which there is no escape (ft)

CH = distance between the weapon and the intersection of a ping bisector and the target cylinder radius at the time of ping transmission (yd)

CR_{jk} = distance from the center of the target cylinder to the projection of the end point of radial (R_{jk}, β_{jk}) in the horizontal plane (yd); $j = J', \cdots, J''$; $k = 1, \cdots, K_j$

c = velocity of sound in the medium (yd/sec)

$\underline{D}$, $\overline{D}$ = value of the index d, respectively below and above search depth, when the boundary nearest search depth of the level L_d is the critical depth (an integer)

$\underline{D}'$, $\overline{D}'$ = upper limits on the numbers of levels L_d respectively below and above search depth (an integer)

D_{MIN} = minimum target depth to be considered (ft)

D_{TM} = maximum target depth (ft)

D_{WS} = weapon search depth (ft)

d = index of the levels L_d above and below search depth (an integer); $d = 1, \cdots, \overline{D}'$ and $d = 1, \cdots, \underline{D}'$ respectively

F(W) = ln $P_k(W)$, (a negative real number)

$F'(W)$ = $F(W) - \lambda\left[g_1(w_1) + g_2(w_2) + g_3(w_3)\right]$, (a negative real number)

$F\ \phi(x)$ = cumulative normal probability distribution with mean zero and standard deviation one evaluated at the argument x (a decimal)

$f_{HD}(x)$ = probability density function of the random variable hit depth evaluated at the argument x (a positive real number)

$f_1(w_1)$ = $\ln P_A(w_1)$, (a negative real number)

$f_2(w'_{2jk}, w''_{2jk})$ = $\ln P_{Hjk}(w'_{2jk}, w''_{2jk})$, (a negative real number);

$j = 1, \cdots, J_k$; $k = 1, \cdots, K$

$f_3(w_3)$ = $\ln P_E(w_3)$ (a negative real number)

G_1 = function relating increased weapon weight to increased weapon speed above S_o (a function of w''_2)

G_2 = function relating weapon speed and depth to fuel expenditure rate (a function of speed and depth)

$g_i(w_i)$ = function relating component weight to volume (a function of w_i; $i = 1, 2, 2', 2'', 3$)

g_i = $P\left[\text{acquisition during the i th ping cycle}\right]$, (a decimal); $i = 1, \cdots, I$

g_{iu} = $P\left[\text{acquisition during the i th ping cycle of the u th search turn}\right]$, (a decimal); $i = 1, \cdots, M$; $u = 1, \cdots, U$

H_j = altitude of an isosceles triangle of area A_j (yd); $J = J', \cdots, J''$

H_k = one way transmission loss at a range of 100k yards (db); $k = 1, \cdots, K'_j$

HD = depth at which the weapon hits the target, a random variable (ft)

$\overline{HD}$ = average hit depth (ft)

$HV_{pq}(w_3)$ = hull vulnerability for kill damage of the p th target surface

area element at the midpoint depth of the q th depth subinterval given an explosive weight w_3 (one or zero); $p = 1, \cdots, P$; $q = 1, \cdots, Q$

$h_i(W)$ = recurrence relation for the allocation model evaluated at the argument W (a negative real number); $1 = 1, 2, 3$

h_n = P [hit on the n th attack (or reattack)] , (a decimal); $n = 1, \cdots, N$

I = upper limit on the number of ping cycles (an integer)

i = index of the total number of ping cycles ($i = 1, \cdots, I$) and of the number of a ping in the first search turn ($i = 1, \cdots, M$), (an integer)

J = a limiting angle in the vertical plane for off-axis signal reception measured from the transducer axis (deg)

J', J'' = smallest and largest angles respectively in the vertical plane at which acquisition may occur (deg)

J_k = the number of combinations of w'_{2jk} and w''_{2jk} considered which sum to w_{2k} (an integer); $k = 1, \cdots, K$

j = index of the planes perpendicular to the vertical plane containing the transducer which are available for acquisition ($j = -J, \cdots, J$) and in which acquisition may occur ($j = J', \cdots, J''$), (an integer); also index of the combinations of w'_{2jk} and w''_{2jk} considered which sum to w_{2k} (an integer) $j = 1, \cdots, J_k$ $k = 1, \cdots, K$

K = number of values of w_{2k} to be considered (an integer)

K_j = the number of radials in plane j used to outline the insonified area (an integer); $j = J', \cdots, J''$

K'_j = the number of ranges in the plane j which have a non-negative available echo excess (an integer) $j = -J, \cdots, J$

k = index on the weapon payload weight used in event 2 ($k = 1, \cdots, K$), on the ranges in the plane j under consideration ($k = 1, \cdots, K'_j$) and on the radials outlining insonified area in plane $j(k = 1, \cdots, K_j)$, (an integer)

$LADV_{du}$ = effective volume element in level L_d at or above the critical depth above search depth on ping zero on the uth search turn (yd^3); $d = 1, \cdots, \bar{D}$; $u = 2, \cdots, U$

$LBDV_{du}$ = effective volume element in level L_d at or below the critical depth below search depth on ping zero on the u th search turn (yd^3); $d = 1, \cdots, \underline{D}$; $u = 2, \cdots, U$

L_d = identification of the d th level (or layer) above and below search depth ($d = 0, \cdots, \bar{D}'$ and $d = 0, \cdots, \underline{D}'$ respectively); also the length of the d th level above and below search depth in the vertical cross-section of the first turn insonified volume of ping zero ($d = 1, \cdots, \bar{D}$ and $d = 1, \cdots, \underline{D}$ respectively), (yd)

M = number of ping cycles in a search turn (an integer)

$MADV_{du}$ = effective volume element in level L_d at or above the critical depth above search depth on ping zero on the u th search turn (yd^3); $d = 1, \cdots, \bar{D}$; $u = 2, \cdots, U$

$MBDV_{du}$ = effective volume element in level L_d at or below the critical depth below search depth on ping zero on the u th search turn (yd^3); $d = 1, \cdots, \underline{D}$; $u = 2, \cdots, U$

$M(BG)_k$ = background masking level at the weapon transducer (db)

MRH = distance traveled by the weapon from time t_o to time t_{CPA} (yd)

m = index indicating the leftmost radial (R_{j1k}, β_{j1k}) or the rightmost radial (R_{j2k}, β_{j2k}) for acquisition at a range of 100k in plane j (one, two)

N = total number of attacks (or reattacks) completed on the target (an integer)

n = index for the attacks (or reattacks) on the target; (an integer) ; $n = 1, \cdots, N$

P = the number of target surface area elements (an integer)

$P[x]$ = probability that event x occurs (a decimal)

P_A = P [target acquisition (event 1)], (a decimal)

P_E = P [kill damage is inflicted on the target by explosion of the weapon warhead (event 3) given that event 2 has occurred] (a decimal)

P_{Ep} = P [hit occurs on the p th target surface area element], (a decimal); $p = 1, \cdots, P$

P_{Eq} = P [hit occurs at the midpoint depth of the q th depth subinterval], (a decimal); $q = 1, \cdots, Q$

P_H = P [target is hit (event 2) given that event 1 has occurred], (a decimal)

P_{Hjk} = P_H for the j th combination of w'_{2jk} and w''_{2jk} making up the weight w_{2k} for event 2; $j = 1, \cdots, J_k$; $k = 1, \cdots, K$

P_k = P [target kill], (a decimal)

PQ_i = P [target is in the i th volume slice of the target volume cylinder at the commencement of search], (a decimal); $i = 1, \cdots, M - 1$

PRR_s = weapon pulse repetition rate for search (pings/sec)

p = index of the target surface area elements (an integer); $p = 1, \cdots, P$

Q = number of hit depth subintervals (an integer)

q = index of the hit depth subintervals (an integer); $q = 1, \cdots, Q$

R = number of values of w_{1r} to be considered (an integer)

R_o = horizontal distance from the weapon to the center of the target volume cylinder (yd)

P_k = P target kill , (a decimal)

R_A = acquisition range (yd)

$RADV_{du}$ = effective volume element in level L_d at or above the critical depth above search depth on ping zero on the u th search turn (yd^3); d = 1, $\cdots$, $\bar{D}$; u = 2, $\cdots$, U

$RBDV_{du}$ = effective volume element in level L_d at or below the critical depth below search depth on ping zero on the u th search turn (yd^3); d = 1, $\cdots$, $\underline{D}$; u = 2, $\cdots$, U

R_I = radius of the target volume cylinder at the time the weapon splashes into the water, (yd)

R_i = radius of the target volume cylinder at the start of the i th ping (yd); i = 1, $\cdots$, M - 1

R_{ijk} = maximum acquisition range at angle β_{jk} in plane j on the i th ping (yd); i = 1, $\cdots$, M - 1; j = J^0, $\cdots J''$; k = 1, $\cdots$, K_j

R_{jk} = length of the k the acquisition radial in plane j (yd); j = J', $\cdots$, J''; k = 1, $\cdots$, K_j

R'_{jk} = length of the projection in the horizontal plane of the k th acquisition radial in plane j (yd); j = J', $\cdots$, J''; k = 1, $\cdots$, K_j

R_{jmk} = length of the acquisition radial (R_{jmk}; β_{jmk}) in plane j (yd); j = J', $\cdots$, J''; m = 1, 2; k = 1, $\cdots$, K'_j

RH = range from the weapon to the target center at time t_o (yd)

r = index of the weapon payload weight used in event 1 (an integer)

r = coordinate of the intersection of the weapon track extension with the vertical plane (A) through the submarine target center perpendicular to any plane containing both target and weapon at time t_o (yd)

r' = coordinate of the intersection of the weapon track with the vertical plane (A') parallel to plane A containing both the submarine target center and weapon at time t_{CPA} (yd)

S = number of values of w_{3s} to be considered (an integer)

S_o = preset minimum value of attack speed (yd/sec)

S_{TC} = target cruise speed (yd/sec)

S_{TM} = target maximum speed (yd/sec)

S_{TS} = target speed during weapon search (yd/sec)

S_{WA} = weapon attack speed (yd/sec)

S_{WS} = weapon search speed (yd/sec)

SL_{jk} = signal level of the target echo at range 100k in plane j (db); $j = -J, \cdots, J$; $k = 1, \cdots, K'_j$

SV_{iu} = effective scanned volume on the i th ping cycle of the u th search turn (yd^3); $i = 1, \cdots, M-1$; $u = 1, \cdots, U$

s = index of the weapon payload weight used in event 3 (an integer); $s = 1, \cdots, S$

T_A = time used by weapon at attack speed during reattack (sec)

T_S = time used by weapon at search speed during reattack (sec)

TES = distance traveled by target during one complete search turn (yd)

TJ = index value of the reference plane j used to define level L_o in the calculation of the special volume elements (an integer)

TS = target strength (db)

TV_i = volume of the i th volume slice of the target cylinder at commencement of weapon ping (yd^3); $i = 1, \cdots, M-1$

t_o = the time weapon sensors lose contact with the target on an attack or reattack (zero reference time)

t_{CT} = time from t_o at which the weapon completes the reattack turn and reacquires the target (sec)

t_{CPA} = time from t_o of closestpoint of approach of weapon and target (sec)

t_{DT} = dead time from t_o at which the weapon commences the reattack turn (sec)

U = maximum number of search turns (an integer)

u = index of the weapon search turns (an integer); $u = 1, \cdots, U$

V = available payload volume (ft^3)

V_j = j th volume element of the insonified volume on ping zero (yd^3); $j = J', \cdots, J''$

v_1 = fuel volume for acquisition, event 1 (ft^3)

v_2 = payload volume for hit, event 2 (ft^3)

v_2' = fuel volume for hit, event 2 (ft^3)

v_2'' = volume for modifications to the propulsion system to provide a maximum (attack) speed capability in excess of S_o (ft^3)

v_{2jk} = j th value of payload volume for event 2 when payload weight for event 2 is w_{2k}; $j = 1, \cdots, J_k$; $k = 1, \cdots, K$

v'_{2jk} = j th value of fuel volume for hit, event 2, when payload weight for event 2 is w_{2k} (ft^3); $j = 1, \cdots, J_k$; $k = 1, \cdots, K$

v''_{2jk} = j th value of volume for attack speed capability when payload weight for event 2 is w_{2k} (ft^3); $j = 1, \cdots, J_k$; $k = 1, \cdots, K$

v_3 = explosive volume (ft^3)

W = available payload weight (lb)

$\underline{W}'$, $\overline{W}'$ = lower and upper limits respectively on the allowable range of fuel weight (lb)

$\underline{W}''$, $\overline{W}''$ = lower and upper limits respectively on the allowable range of weight for modifications to the propulsion system to provide a maximum speed capability in excess of S_o (lb)

$\underline{W}'''$, $\overline{W}'''$ = lower and upper limits respectively on the allowable range of explosive weight (lb)

WEL_k = weapon enabling level at a range 100k yards (db)

WS = weapon on-axis source level (db at 1 yard from transducer)

w_1 = fuel weight for acquisition, event 1 (lb)

w_{1r} = r th value of acquisition fuel weight (lb); $r = 1, \cdots, R$

w_2 = payload weight for event 2 (lb)

w_{2k} = k th value of payload weight for event 2 (lb); $k = 1, \cdots, K$

w_2' = fuel weight for hit, event 2 (lb)

w_2'' = weight for modification to the propulsion system to provide a maximum (attack) speed capability in excess of S_o (lb)

w_{2jk}' = j th value of fuel weight for hit, event 2, when payload weight for event 2 is w_{2k} (lb); $j = 1, \cdots, J_k$; $k = 1, \cdots, K$

w_{2jk}'' = j th value of weight for attack speed capability when payload weight for event 2 is w_{2k} (lb); $j = 1, \cdots, J_k$; $k = 1, \cdots, K$

w_3 = explosive weight (lb)

w_{3s} = s th value of explosive weight (lb); $s = 1, \cdots, S$

w_{0t} = average weapon fuel expenditure rate in dive to search depth (lb/sec)

w_{1t} = weapon fuel expenditure rate at search depth and speed (lb/sec)

w_{2t} = weapon fuel expenditure rate at attack speed and at the average depth for the attack (lb/sec)

w_{Atq} = weapon fuel expenditure rate at attack speed and at the mid-point of the q th depth interval (lb/sec); q = 1, ..., Q

w_{Stq} = weapon fuel expenditure rate at search speed and at the mid-point of the q th depth interval (lb/sec); q = 1, ..., Q

w_{OSD} = fuel used in dive to search depth (lb)

w'_{2n} = fuel used in the n th attack or reattack (lb)

α ,α' = limiting angles for off-axis reception in the azimuth plane, to left and right respectively of the weapon transducer longitudinal axis at time of reception (deg)

β' = angle in plane zero the right hand edge of a ping makes with the reference axis after the overlap correction (deg)

β_{jk} = azimuth angle of the k th radial in plane j (deg); $j = J', \cdots, J''$; $k = 1, \cdots, K_j$

β_{jmk} = azimuth angle of the acquisition radial (R_{jmk}, β_{jmk}) in plane j (deg); $j = -J, \cdots, J$; $m = 1, 2$; $k = 1, \cdots, K'_j$

Δw = increment of weight of any of the weight variables (lb)

δ = angle through which the weapon turns between pings (deg)

δ' = angle assigned to the M th ping (deg)

λ = Lagrange multiplier (non-negative real number)

μ = weapon angle of attack (pitch), (deg)

σ, σ' = standard deviations of the normally distributed random variables r and r' respectively (yd)

ϕ_{DS} = angle of dive to search depth (deg)

ϕ_k = angle through which the weapon turns while a ping travels to a range of 100k yards and returns (deg)

ω_R = weapon reattack turn rate (deg/sec)

ω_S = weapon search turn rate (deg/sec)

CHAPTER I

INTRODUCTION

The Problem

An air-dropped anti-submarine torpedo may be characterized by a long list of parameters. Specification of parameter values in a specific weapon is made such that the weapon may achieve a maximum of effectiveness in an actual tactical situation in wartime. The state of the art in weapon design will determine the values of many of the weapon parameters. For example, the transducer parameters may be prespecified by the status of some continuing torpedo sonar development program. Cost considerations may also be of great importance in the evaluation of alternative proposals for the design parameter values. For example, a choice between two propulsion systems may hinge on a difference in development costs necessary to produce a reliable weapon. However, for torpedoes and more especially for air-dropped torpedoes, the restrictions of weight and of physical dimensions (or volume) resulting from the varied mix of capabilities of the prospective weapon delivery systems are also very significant in the weapon design.

Consider the restraints of weight and volume. One of the design parameters which has a great effect on weapon weight and volume is the maximum operating depth. An increase of a few hundred feet in the operating depth requirement will require a more substantial structure. This in turn may result in a decreased weight and volume to be made available for other weapon components. However, intelligence reports or estimates of the depth capabilities of prospective

submarine targets in the time period for which the weapon is to be designed may prompt the decision maker to fix the design operating depth as an input to the design problem. Similarly the advance specification of a minimum attack speed capability may be prompted by intelligence estimates and will certainly act to place an upper limit on weight and volume available for other weapon components. Suppose that a study of specifications from higher authority, engineering capabilities to produce the various weapon components, cost considerations or some appropriate optimization procedures have resulted in specification of values for all weapon design parameters except

1. Amount of fuel
2. Amount of explosive
3. Maximum speed (above some preset minimum value).

Further, suppose the restraints on weight and volume have been determined such that if the amount of fuel and explosive and the maximum speed capability above some specified minimum are considered as a weapon payload, then the available payload weight and volume are provided to the decision maker. The decision problem of allocation of available payload weight and volume to amount of fuel, amount of explosive, and to maximum speed capability above some specified minimum is the problem considered in this paper.

The Approach

An initial approach to the allocation problem is to set certain minimum limits on the parameter values. The minimum value of maximum (attack) speed has been specified. There must be sufficient fuel to provide some minimum probability of target acquisition. (In the case of continuous circular search at a constant preset depth as is considered in this paper a minimum probability of acquisition might result from a dive to search depth and completion of one complete search turn.) Then there must be sufficient fuel for at least one attack under some assumed conditions of acquisition range and target course and speed. Finally there must be at least enough explosive to result in kill damage if a hit occurs at the most favorable depth and point of contact on the target surface. Having determined these allocation minimums there remains the problem of allocating the balance of available payload weight and volume.

A second step may determine a maximum value of attack speed. This maximum may be determined from a study including the probable operating depth, the characteristics of the guidance system, and the generated self-noise level and its effect on the ability of the sonar to maintain contact with the target on the attack run.

What is the next step? How does the decision maker decide between enough additional fuel for one reattack and enough additional explosive to produce kill damage over a given additional element of target surface? What measure of effectiveness is used?

This paper proposes a procedure which may provide the decision maker with a quantitative analysis or comparison of the tradeoffs

involved in alternatives of allocation among fuel, explosive, and additional attack speed capability. The tradeoffs are measured in terms of probability of kill as a measure of effectiveness. Probability of kill is dependent upon weapon reliability and upon the capability of the enemy to employ countermeasures (in addition to maneuvering). It is assumed in this paper that weapon reliability and enemy capability to employ countermeasures (in addition to maneuvering) are both independent of the amount of fuel and explosive and of the weapon attack speed within the range of values between the preset minimum and some determinable maximum. This enables removal of reliability and enemy countermeasures capability (less maneuvering) from the problem and probability of kill may then be factored into three independent probabilities; those of probability of acquisition, probability of hit given acquisition, and probability of kill given hit. An optimization is made of probability of kill as a function of the fuel, explosive, and additional attack speed capability with weight and volume restraints. This does not provide a number for use as an operational value of kill probability. However the dynamic programming methods used provide a solution which specifies the optimal allocation of weight and volume.

It is emphasized that a quantitative comparison of the tradeoffs involved in the allocation process may be obtained only by some common analysis of the three study variables as they act upon the events of acquisition, hit, and kill.

The dynamic programming optimization model developed generates an input requirement for (1) probability of acquisition as a function of available fuel, (2) probability of hit as a function of available fuel and attack speed and (3) probability of kill damage as a function of explosive weight. These requirements may be satisfied in a number of ways including

1. analytical derivations in terms of obtainable or estimable inputs
2. simulation (war gaming)
3. extrapolation of empirical data from operational or development tests
4. a combination of the above

The model user may choose any of these methods so long as the resulting inputs are compatible with the allocation model. However, any of the methods used will require an assumption about the tactical situation with respect to which the optimization is to be made. Consider the actual tactical situation in which the weapon will be used. Assume that methods exist to determine the optimum values of all weapon parameters, given that the weapon will be used in a completely specified tactical situation. The optimum parameter values determined will be different for each different tactical situation considered. The space of different tactical situations is infinite. However a single set of parameter values must be specified for the weapon, ultimately to be produced in quantity. By what criteria is the specific tactical situation to be selected for use in the optimization of the weapon parameters? The probability

that the weapon will actually be used in the tactical situation selected is zero. The following two criteria will be used in this paper in the analytical derivation of the three required probabilities.

Criterion 1: Select an unfavorable tactical situation which is not extreme. This may be thought of as a modified 'maximin' criterion. A maximin criterion implies optimization of the weapon parameters to maximize effectiveness for that tactical situation which offers minimum probability of weapon success. The extreme situation is modified in this paper because the low probability of kill resulting from assumption of an unfavorable situation in the extreme would undoubtedly distort the relationships between the study variables which would apply in a greater percentage of cases. In general a weapon which is optimized for an unfavorable situation will be able to cope with a more favorable situation for which its parameter values were not optimized.

Criterion 2: Select a tactical situation which is sufficiently simplified as to be within the scope of this paper. Progress in work of this kind is more often made by starting with something relatively simple and adding sophistication after experience is gained with the model.

Relation of this work to previous work

The allocation problem has been studied before by R. C. Brumfield in references 1 and 2 in which algebraic expressions were derived relating total weight and volume of a weapon of arbitrary speed and range to performance characteristics and component characteristics. This enabled determination of optimum running speed and range for stern chase vehicles under conditions of minimum propulsion system weight, minimum total weight of propulsion system and associated shell, and minimum propulsion system volume. The approach in this paper is essentially different from that of references 1 and 2 although the problem is much the same. The previous work optimized total weight and volume and attack speed for the event of a single attack run, given explosive weight, a length of run in the acquisition phase, and acquisition range. This paper optimizes an allocation of payload weight and volume to fuel, explosive, and attack speed for the events of acquisition, hit (attack and reattacks) and kill given the maximum allowable payload.

A great deal of work has been done in the study of various phases of the torpedo problem. References 3 and 4 form the background on which the development in Chapter III is based. Reference 4 develops a continuous volume scan rate for a single circular search turn. In this paper, in the first three sections of Chapter III, a volume scan for a single ping on the first search turn is developed. This is a modification of the continuous scan rate to the discrete case where the discrete interval is a single ping cycle. However, the

development in this paper is continued to obtain a probability of acquisition not only for a single circular search turn but for continuous turns until fuel exhaustion. This further development is made possible because a tactical situation is postulated in accordance with criteria one and two. The postulation is Chapter III limits the usefulness of the specifics of the development to the search pattern considered but is required in order to obtain the probability of acquisition input to the allocation model. The logic of the development may be modified as appropriate to provide for other postulated tactical situations in accordance with requirements to study a particular search pattern. The study of search turns subsequent to the first and until fuel exhaustion is necessary to provide probability of acquisition as a function of fuel weight throughout the range of the argument.

The development in Chapter IV of probability of a hit in any given pass at the target has no known precedent in the literature and is presented as an alternative way of looking at the hit probability problem which may be of interest.

It is emphasized that Chapters III, IV, and V are samples of possible methods of attacking the problem of finding inputs to the allocation model of Chapter I which are specifically designed to permit examination of the tradeoffs between alternative allocations of payload weight and volume among the three study variables.

Structure of the paper

Chapter I contains the development of a mathematical model of the torpedo problem in a form such that the dynamic programming techniques of Chapter II of reference 5 may be applied to solve the allocation decision problem. Chapters III, IV, and V contain the development of the three required inputs to the allocation model of Chapter II: probability of acquisition, probability of hit given acquisition and probability of kill given hit. Appendix A is a description of the computer flow chart notation to be used in the flow charts of the other appendices. Appendix B is the computer flow chart of the dynamic programming allocation model of Chapter I. Appendices C 1 through C 8 contain the mathematical details of the developments in sections I through VIII of Chapter III plus the corresponding computer flow charts. This mathematical supplement to Chapter III is included in appendices to enable emphasis of the logic flow in Chapter III with a minimum of mathematical detail. Appendix D contains the computer flow chart for Chapter IV.

Summary

A mathematical model is provided which enables optimization of the decision problem of the allocation of available weight and space in an air-dropped anti-submarine torpedo among three study variables: amount of fuel, amount of explosive and attack speed capability above some preset minimum. The model is restricted to the study of alternative allocations which do not differ with respect to their effect on weapon reliability and to their effect on enemy countermeasures capability (other than maneuvering capability.)

Mathematical models are developed which relate (1) probability of acquisition to endurance for the case of circling search at constant depth, (2) probability of hit given acquisition to endurance and attack speed, (3) probability of hit on a close-in attack run to attack speed, and (4) probability of kill damage given a hit to amount of explosive used. These four models are developed as specific examples of inputs needed to be compatible with the requirements of the allocation model. Computer flow charts are provided for each model except the last to facilitate programming in any desired computer language and also as an alternative method of study of the development of the model logic.

CHAPTER II

THE ALLOCATION MODEL

Consider the design of an air-dropped anti-submarine torpedo such that the amounts of fuel and explosive to be included and the capability of attack speed in excess of some predetermined minimum value may be considered together as a payload for which there is available a predetermined maximum of weight and volume. Consider the employment of the torpedo from the time it splashes into the water until the time it explodes in the proximity of a submarine target as an ordered sequence of the following three events each of which must occur in succession for accomplishment of the torpedo mission:

1. Acquisition after a search run following some preset pattern
2. Hit after an attack (and if necessary a series of reattacks)
3. Kill damage resulting from explosion of the weapon in the proximity of the target

Divide the study variable, amount of fuel, into two separate and independent study variables:

1. Amount of fuel to be used in the event of acquisition (event 1)
2. Amount of fuel to be used in the event of hit (event 2)

Restrict the study variable, attack speed, to a range of values such that

1. The lower limit of the range is some preset minimum value which is greater than the given value of weapon search speed.
2. As the attack speed is allowed to vary over its range, the fuel expenditure rate at attack speed and at any depth under considera-

tion may also vary. However as the propulsion system is modified to allow for changes in the value of its maximum (attack) speed capability, the fuel expenditure rate at search speed and depth remains approximately constant.

Make the following assumptions:

Assumption 1: Weapon reliability and target capability to employ countermeasures other than maneuvering are both independent of changes (within the ranges to be considered) in the values assigned to the four study variables:

1. Amount of fuel for event 1
2. Amount of fuel for event 2
3. Amount of explosive
4. Weapon attack speed

Assumption 2: The weapon is 100% reliable.

Assumption 3: The enemy target submarine has no countermeasures capability except that of maneuvering.

Note: Assumptions 3 and 4 are made in order to justify the relationship between the study variables and the measure of effectiveness to be introduced later. The fact that they are intuitively incorrect does not detract from the validity of the optimization in the model because of assumption 1.

Assumption 4: The three events of acquisition, hit and kill are independent events in a probabilistic sense. That is, if

$P[x]$ = Probability that event x occurs

$P_A = P[\text{target acquisition (event 1)}]$

$P_H = P[\text{target is hit (event 2) given that event 1 has occurred}]$

$P_E = P[\text{kill damage is inflicted on the target by explosion of the weapon warhead (event 3) given that event 2 has occurred}]$

If $P_k = P[\text{target kill}]$

then $P_k = P_A \cdot P_H \cdot P_E$

Define:

W = available payload weight (lb)

V = available payload volume (ft^3)

w_1 = fuel weight for acquisition, event 1 (lb)

w_2' = fuel weight for hit, event 2 (lb)

w_2'' = weight for modifications to the propulsion system to provide a maximum (attack) speed capability in excess of some preset minimum value (lb)

S_o = preset minimum value of attack speed (yd/sec)

w_3 = explosive weight (lb)

v_1 = fuel volume for acquisition, event 1 (ft^3)

v_2' = fuel volume for hit, event 2 (ft^3)

v_2'' = volume for modifications to the propulsion system to provide a maximum (attack) speed capability in excess of S_o (ft^3)

v_3 = explosive volume (ft^3)

w_2 = payload weight for hit, event 2 (lb)

$w_2 = w_2' + w_2''$

v_2 = payload volume for hit, event 2 (ft^3)

$v_2 = v_2' + v_2''$

Δw = increment of weight of the any of the weight variables, all of which are varied in increments of equal size (lb)

$g_i(w_i)$ = the relationship between the i th weight variable and the i th volume variable, $i = 1, 2', 2'', 3$

$g_i(w_i) = v_i$; $i = 1, 2', 2'', 3$; $g_1(w) = g_2'(w)$

$f_1(w_1)$ = natural logarithm (ln) of P_A expressed as a function of w_1

$f_2(w_2', w_2'')$ = ln P_H expressed as a function of w_2' and w_2''

$f_3(w_3)$ = ln P_F expressed as a function of w_3

$F = \ln P_k = \sum_{i=1}^{3} f_i$

$\underline{W}', \bar{W}'$ = lower and upper limits respectively on the allowable range of fuel weight (lb)

$\underline{W}'', \bar{W}''$ = lower and upper limits respectively on the allowable range of weight for modifications to the propulsion system to provide a maximum speed capability in excess of S_o (lb)

$\underline{W}''', \bar{W}'''$ = lower and upper limits respectively on the allowable range of explosive weight (lb)

The concept of the lower and upper limits on the ranges of the study variables is as follows. It will be obvious to the design engineer that very small values or either fuel or the explosive weights will result in a negligible probability of success. Thus there is no need to examine the vicinity of the extreme values of zero and W. However the values of the limits may be chosen as close to zero and W as is desired. In the case of $\underline{W}''$, the value

chosen may well be zero as this implies that the considered minimum of attack speed is to be S_o, the preset minimum. In the case of $\bar{W}''$ the value chosen will probably be well under W as the maximum of the range of values of attack speed is limited by considerations of resultant guidance problems, excessive self-noise levels, and so forth. The hit event, event 2, contains two study variables which must be considered together. For events 1 and 3 only one study variable is considered per event (or programming stage). Thus for each value of fuel weight (w_1) there is associated a single value of acquisition probability $P_A(w_1)$ and a single value of volume $v_1 = g_1(w_1)$. Similarly for values of explosive weight w_3.

However

$$w_2 = w_2' + w_2''$$
$$v_2 = v_2' + v_2'' = g_2'(w_2') + g_2''(w_2'')$$

This requires that for each value of weight (w_2) there be an associated set of values of hit probability $P_H(w_2)$ and of volume v_2, one value for each combination of values of w_2' and w_2'' which sum to the considered value of w_2. A suboptimization of w_2' and w_2'' to provide a single optimal value of $P_H(w_2)$ for each w_2 as an input to the allocation model would neglect the associated volume restraints. Therefore index the weight w_2 by k($k = 1, \cdots, K$) over its range

$$\min \left\{ \underline{W}', \underline{W}'' \right\} \leq w_2 \leq \min \left\{ \bar{W}' + \bar{W}'', W \right\}$$

and define

w_{2k} = k the value of weight to be distributed between w_2' and w_2''; k = 1, ···, K (1b)

w'_{2jk} = j th value of w_2' associated with w_{2k}; j = 1, ···, J_k (1b)

w''_{2jk} = j th value of w_2'' associated with w_{2k}; j = 1, ···J_k (1b)

P_{Hjk} = j th value of P_H associated with w_{2k}; j = 1, ···, J_k

v'_{2jk} = j th value of v_2' associated with w_{2k}; j = 1, ···, J_k; k = 1, ···, K

V''_{2jk} = j the value of v_2'' associated with w_{2k}; j = 1, ···, J_k;

$v_{2jk} = v'_{2jk} + v''_{2jk}$ = j the value of v_2 associated with w_{2k}; j = 1, ···, J_k (ft^3)

The available payload W and the upper and lower weight limits or the weight increment Δw are so chosen that all the weights are divisible by Δw. Then there will be

$$\frac{\bar{W}'}{\Delta w} - \frac{\underline{W}'}{\Delta w} = R$$

considered values of w_1 denoted w_{1r}; r = 1, ···, R,

there will be

$$\frac{\min\left\{\bar{W}' + \bar{W}'', W\right\}}{\Delta w} - \frac{\min\left\{\underline{W}', \underline{W}''\right\}}{\Delta w} = K$$

considered values of w_2 denoted w_{2k}; k = 1, ···, K

and there will be

$$\frac{\bar{W}'''}{\Delta w} - \frac{\underline{W}'''}{\Delta w} = S$$

considered values of w_3 denoted w_{3s}; $s = 1, \cdots, S$

The functions $g_1(w_1)$, $g_2'(w_2')$, $g_2''(w_2'')$, $g_3(w_3)$, $f_1(w_1)$, and $f_3(w_3)$ are assumed to be non-decreasing. That is, an increase in the argument will not decrease the value of the function. In the case of $f_2(w_2', w_2'')$ it is assumed that any increase in either of the arguments while maintaining the other fixed will not decrease the value of the function.

The problem is to maximize $F = \ln P_k$

subject to

$$w_1 + w_2 + w_3 \leq W$$
$$v_1 + v_2 + v_3 \leq V$$

Consider the new problem to maximize

$$F' = F - \lambda \left[g_1(w_1) + g_2'(w_2') + g_2''(w_2'') + g_3(w_3) \right]$$

subject to

$$w_1 + w_2 + w_3 \leq W$$
$$g_i(w_i) \geq 0; \; i = 1, 2', 2'', 3$$

where λ is a non-negative real number (Lagrange-multiplier).

Define the recurrence relations:

$$h_1(W) = f_1(W) - \lambda g_1(W) \qquad \text{if } W = w_{1r}; \; r = 1, \cdots, R$$

$$= -\infty \qquad \text{otherwise}$$

$$h_2(W) = \max_{w_{2k};\ k = 1, \cdots, K} \left\{ \max_{1 \le j \le J_k} \left[f_2(w'_{2jk}, w''_{2jk}) - \lambda v_{2jk} \right] + h_1(W - w_{2k}) \right\}$$

where

$$f_2(w'_{2jk}, w''_{2jk}) = \ln P_{Hjk} = \ln F_H(w'_{2jk}, w''_{2jk})$$

$$v_{2jk} = g'_2(w'_{2jk}) + g''_2(w''_{2jk})$$

$$h_3(W) = \max_{w_{3s};\ s = 1, \cdots, S} \left[f_3(w_{3s}) - \lambda g_3(w_{3s}) + h_2(W - w_{3s}) \right]$$

The dynamic programming methods of Chapter II of reference 5 are next applied to the recurrence relations h_1, h_2, and h_3 for some fixed λ to obtain a solution for the optimum value of $h_3(W) = F'$ and for the corresponding optimum values of w_1, w'_2, w''_2, and w_3.

Next the corresponding optimum values of v_1, v_2, and v_3 are computed using the computer determined optimum values of w_1, w'_2, w''_2, and w_3. The sum $(v_1 + v_2 + v_3)$ is then compared with the available payload volume V. Probably $v_1 + v_2 + v_3 \neq V$.

Then if

$$v_1 + v_2 + v_3 > V$$

the value of λ must be increased and the dynamic program repeated until the minimum value of λ is found which results in

$$v_1 + v_2 + v_3 \leq V$$

If $v_1 + v_2 + v_3 < V$

the value of λ is decreased until the minimum value of λ is found which results in

$$v_1 + v_2 + v_3 \leq V$$

The optimum value of $h_3(W)$, w_1, w_2', w_2'' and w_3 determined for such a λ specify the solution values of the three study variables.

The Lagrange-multiplier, λ, may be thought of as a cost coefficient. If: $\lambda = o$

$$v_1 + v_2 + v_3 \leq V$$

then the volume restraint may be thought of as having cost nothing. That is, the measure of effectiveness has been optimized solely with respect to the weight restraint and the result has met the volume requirements. On the other hand, if $\lambda = o$

$$v_1 + v_2 + v_3 > V$$

then some positive value must be assigned to λ which will result in meeting the problem volume restraint $v_1 + v_2 + v_3 \leq V$.

Ultimately the subsequent process of optimization with some positive value of λ will result in smaller optimum values of one or more of the weight variables with a correspondingly reduced value of the measure of effectiveness.

This reduction in the solution value of the measure of effectiveness may be considered as the cost of having a volume restraint. Of course if weights and volumes are interchanged in the model the resulting λ will be a measure of the cost of the weight restraint. If the solution value of probability of kill is desired then

$$P_k = \exp\left[h_3(W) + \lambda(v_1 + v_2 + v_3)\right]$$

where $h_3(W)$, v_1, v_2 and v_3 are solution values.

The computer flow chart to accomplish this dynamic programming procedure is contained in Appendix B.

It may be of interest to allow the values of the available payload weight and volume (W and V) to vary in the vicinity of their preset values. Then the sensitivity of the effectiveness measure, P_k, to such small variations may be examined in order to determine the possible value of modifications to the weapon delivery systems to allow for a different (more effective) load capability.

Input Requirements for the allocation model

W = Available payload weight (lb)

V = Available payload volume (ft^3)

$\underline{W}'$, $\overline{W}'$ = lower and upper limits respectively on the allowable range of fuel weight (lb)

$\underline{W}''$, $\overline{W}''$ = lower and upper limits respectively on the allowable range of weight for modifications to the propulsion system to provide a maximum (attack) speed capability in excess of a minimum preset value (lb)

$\underline{W}'''$, $\bar{W}'''$ = lower and upper limits respectively on the allowable range of explosive weight (lb)

$g_1(w_1)$; i = 1, 2'', 3 = the relationships between the i th weight variable and the i th volume variable; i = 1, 2'', 3

$P_A(w_1)$ = probability of target acquisition expressed as a function of the fuel weight for acquisition

$P_H(w_2', w_2'')$ = probability of hitting the target given that target acquisition has occurred expressed as a function of the fuel weight for hit and of the weight for modifications to the propulsion system to provide a maximum (attack) speed capability in excess of S_o

$P_E(w_3)$ = probability of target kill given that a hit has occurred expressed as a function of the explosive weight

S_o = minimum preset value of attack speed (yd/sec)

Δw = size of the increments of weight of the weight variables, all of which are varied in increments of equal size (lb)

The payload weight and volume are determined by the capabilities of the weapon delivery systems. The limits of the weight variables are determined by a preliminary study which reflects limiting design considerations and reduces the possible ranges of the weight variables to reduce computer time. The weight-volume relationships are determined by engineering considerations reflecting the state of the art. The minimum attack speed is determined by intelligence estimates or CNO requirements. The size of the weight increments

is determined by considerations of required accuracy of results and available computer time. The probabilities of acquisition, hit and kill may be obtained in a number of ways including

1. analytical derivations in terms of obtainable or estimable inputs
2. simulation (war gaming)
3. extrapolation of empirical data from operational or development tests
4. a combination of the above

The remaining chapters provide samples of possible analytical methods of providing the three required probability inputs which are specifically designed to be compatible with the allocation model requirements.

It is possible to extend the model to provide for cost restraints. The procedure would require a second Lagrange-multiplier in the function to be maximized (F'). Additional inputs required would be the relationsuips between the costs and the corresponding weight variables. It is considered, however, that cost restraints are not significant to the problem considered in this paper. Variations in the amount of fuel and amount of explosive have little effect on the costs involved. Even changes in attack speed within the limits appropriate for consideration will probably have negligible cost effect when compared to the total cost of developing, producing and operating a weapon in the course of its life cycle.

CHAPTER III

PROBABILITY OF ACQUISITION

Consider a tactical situation in which a weapon is delivered by some air delivery weapons system to a splash point on the surface of the water in the vicinity of a submarine target and make the following assumptions.

Assumption 1: The splash point defines the center of the top of a right circular cylinder called the target volume cylinder. The probability is quite small and is assumed zero that the target is outside of the target volume cylinder. The target is placed at random within the target volume cylinder.

Assumption 2: The weapon dives to its assigned search depth on a straight line track and commences a right circular search turn at constant depth, speed, and turn rate immediately upon reaching search depth. The weapon searches in the active acoustic mode commencing its first ping upon commencement of the search turn. The weapon continues its turn until target acquisition or fuel exhaustion.

Assumption 3: The weapon does not acquire a false target.

Assumption 4: The search turn of the weapon is approximated by rotating the weapon about a vertical axis through the point at which the weapon commences search.

Assumption 5: While the weapon is diving to search depth, the radius of the target volume cylinder increases at a rate equal to the target cruise speed.

Assumption 6: The target is alerted on commencement of the weapon search turn by the first 'ping' of the weapon. Thereafter the target opens at constant depth and speed radially from the projection of weapon position on the horizontal plane through the target.

Assumption 7: During search the radius of the target volume cylinder increases at a constant rate equal to the target speed.

Assumption 8: The 'definite range' law as developed in Chapter II of reference 6 applies to acquisition. Thus on a given radial during a ping cycle under specified conditions there is a range beyond which acquisition probability is zero and within which acquisition probability is one. Regions closer than the 'definite range' limits are considered scanned.

Assumption 9: During search the weapon makes a series of independent glimpses for the target, one for each ping cycle.

Assumption 10: Target cylinder volume is considered constant during each ping cycle at the value valid at the initiation of the ping.

Assumption 11: The water medium is considered to be isotropic. That is, all environmental parameters that affect acoustic performance have a constant value throughout the considered volume.

Assumption 13: Consider planes containing the transducer and angles measured at the transducer. Consider the vertical plane containing the weapon longitudinal axis as the vertical reference plane. As the weapon sweeps through an angle ϕ in the horizontal plane it sweeps through a lesser angle ϕ' in a plane perpendicular to the vertical reference plane but inclined to the horizontal. Also if the inclination of the plane inclined to the horizontal as measured in the vertical reference plane is α then the inclination of the plane inclined to the

horizontal as measured in a vertical plane other than the reference plane is a lesser angle α'. In this paper it is assumed that

$$\phi' = \phi$$

$$\alpha' = \alpha$$

In practical situations likely to be considered, this assumption results in much smaller errors than those created by treating the range problem in discrete steps as is done in this chapter.

Define:

$P_A = P_A(I) = P\left[\text{acquisition in I ping cycles}\right]$

where $P\left[X\right]$ = probability of event X happening

$g_i = P\left[\text{acquisition during the i th ping cycle}\right]$; i = 1, ···, I

w_{lt} = weapon fuel expenditure rate at search depth and speed (lb/sec)

PRR_s = weapon active sonar pulse repetition rate for search (pings/sec)

w_{OSD} = fuel used by weapon to reach search depth. (lb)

w_l = fuel available for acquisition. (lb)

Then, as developed in Chapter 2 of reference 6,

$$P_A = 1 - \prod_{i=1}^{I} (1-g_i)$$

and since

$$\left[\left[\frac{(w_l - w_{OSD})\, PRR_s}{w_{lt}}\right]\right] = I$$

and $\left[\left[X\right]\right]$ = greatest integer $\leq X$ and where I is the number of complete ping cycles available for the event of acquisition, it may be said that P_A is a function of w_l and a table may be computed of P_A vs w_l where w_l is allowed to vary between $\underline{W}'$ and $\bar{W}'$.

Of course if I is less than 1 then P_A is defined to be zero. The probability of acquisition in a specified glimpse (g_i) may be considered as a coverage factor. The volume containing the target with positive probability is a region to be covered. A portion of the region to be covered is scanned (covered) by the weapon in accordance with the definite range law. Because the target has been randomly placed in the target volume cylinder (region of target uncertainty to be covered) then

$$g_i = \frac{\text{volume scanned on i th ping cycle}}{\text{target volume during i the ping cycle}}$$

Thus the problem is reduced to finding what portion of the volume of target uncertainty is scanned by the weapon on any ping. This will be different for every ping.

The following development for P_A is divided into eight consecutive steps. The development is amplified as appropriate and complete flow charts for a computer program are provided in appendices C 1 through C 8.

1. Available Echo Excess (B_{jk})

Consider a vertical plane containing the longitudinal axis of the weapon at the time of ping transmission. Consider, as in Figure 1, a second plane perpendicular to the first and containing the weapon transducer such that it makes an angle j with the weapon longitudinal axis. A set of such planes is considered such that the angle j is associated with the j th plane. In the vertical plane angles are measured positive counterclockwise from the reference axis.

Design or operational considerations will make it necessary to prohibit reception of signals from off the transducer axis in the vertical plane in excess of some limting angle J, not greater than 90 degrees. Allowing one degree between each of the considered planes j, the index j will assume the range of values $-J, -J+1, \cdots, -1, 0, 1, \cdots, J$. Thus 2J+1 planes are to be considered, one containing the transducer axis and J planes on either side.

Consider increments of target range of 100 yards in the plane j. For an assumed target positioned in the j th plane at a range of 100k yards on the transducer azimuth axis projection in the plane j (point P in Figure 1), there exists an echo excess (B_{jk}) by which the signal level (SL_{jk}) of the echo reflected from the target exceeds the background masking level at the weapon transducer ($M(BG)_k$). That is:

$$B_{jk} = SL_{jk} - M(BG)_k; \quad j = -J, \cdots, J; \quad k = 1, \cdots, K_j^0$$

Assuming a definite range law of acquisition, a negative echo excess yields zero probability of acquisition and a positive echo excess yields a probability of acquisition of one. The maximum range of acquisition on

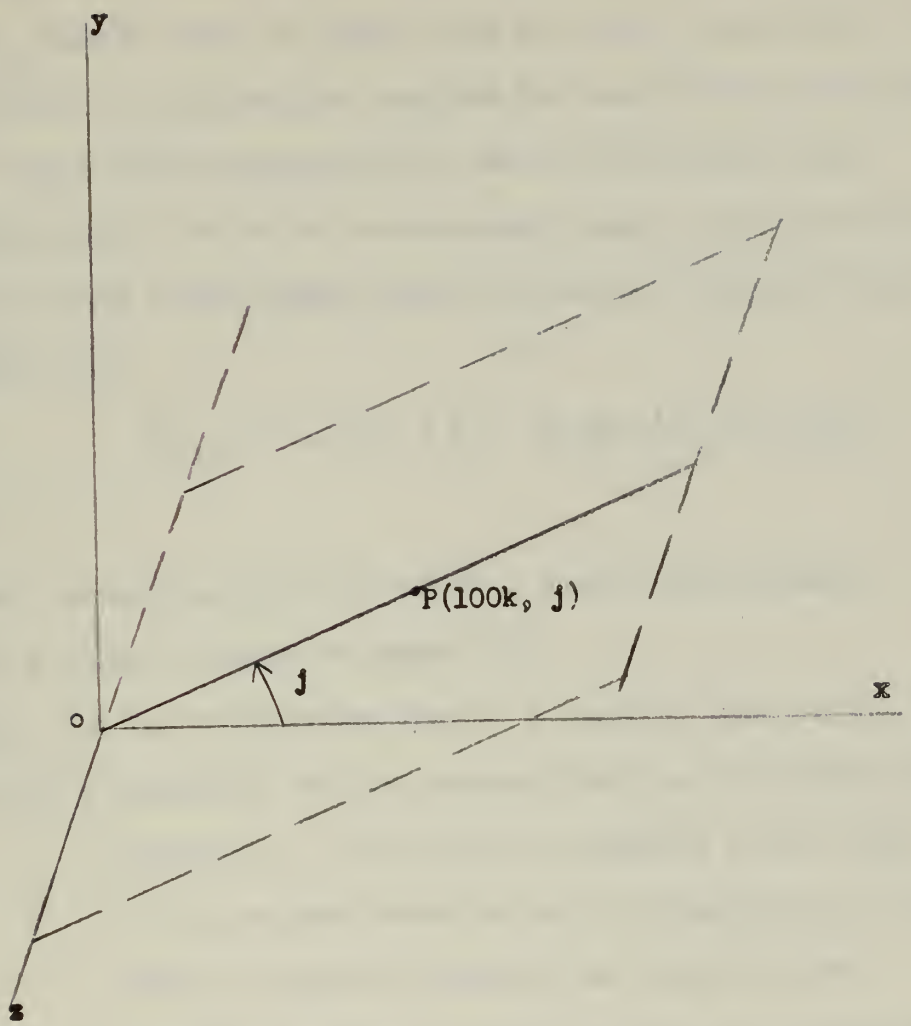


Figure 1: The plane j. The transducer is at the origin pointing in the x direction. Line ox is the transducer axis extension. Lines ox, oP, and oy are in the vertical plane. Lines oP and oz are in the plane j. The polar coordinates of the point P in the vertical plane using the x axis as a reference are (100k, j).

the azimuth axis in the j th plane is thus the range of $100K'_j$ yards at which B_{jk} equals zero. At ranges less than $100K'_j$ yards there is an echo excess of B_{jk} above that required for acquisition on the azimuth axis. This excess is available for use off the aximuth axis.

The signal level varies with assumed target range and with the elevation angle of the target above or below the tranducer axis in accordance with

$$SL_{jk} = WS + TS - 2\,H_k + 10 \log \left[b_1(j)\, b_2(j) \right]$$

where:

WS = on-axis source level (db at 1 yard from transducer)

TS = target strength of target (db)

H_k = one way transmission loss at a range of 100k yards (db)

$b_i(j)$ = normalized vertical pattern function for transmitting pattern (i.e., the ratio of intensity at the angle j off the vertical axis but on the azimuth axis to intensity on-axis in both the vertical and aximuth planes.

$b_2(j)$ = normalized vertical pattern function for receiving pattern.

There are several different kinds of masking noises and it is their total effect which is considered as the interference background masking level. Included are self-noise, target noise, ambient noise, and volume and boundary reverberation.

There is also an enabling level designed into the weapon electrical circuitry to protect the acoustic system from excessive interference and to avoid interpretation of the interference as a signal. As used

here, the weapon enabling level (WEL_k) includes the weapon recognition differential. It is a function of range (or time) and decreases with increasing range. The background masking level ($M(BG)_k$) is the larger of the two: interference background masking level and the weapon enabling level.

The output of this step is a table of the available echo excess (B_{jk}; $j= -J, \cdots, J$; $k= 1, \cdots, K'_j$).

2. Scanned Volume Radials: General Case

In each of the planes j there is now defined for each value of range (100k) an available echo excess B_{jk} for acquisition on the azimuth axis. This available excess may be used to find limits for off-axis acquisition. Define an angle ϕ_k to be the angle through which the weapon turns while a ping travels to a range of 100k and returns.

Then

$$\phi_k = \frac{200k\omega_S}{c}$$

where

ω_S = weapon search turn rate (deg/sec)

c = velocity of sound in the medium (ft/sec)

For echo ranging in the plane j to a range of 100k, an energy pulse transmitted at angle β will return at the angle $\beta - \phi_k$.

The echo excess $A_{jk}(\beta)$ required to acquire the target at range 100k in plane j at angle β off the aximuth axis is

$$A_{jk}(\beta) = -10 \log b_3(\beta) - 10 \log b_4(\beta - \phi_k)$$

where $b_3(\beta)$ = normalized aximuth pattern function for transmitting pattern

$b_4(\beta)$ = normalized aximuth pattern function for receiving pattern

If the required echo excess $A_{jk}(\beta)$ exceeds the available echo excess B_{jk} then acquisition will not take place at range 100k at an azimuth angle removed β degrees from the reference axis of plane j. However if $A_{jk}(\beta)$ does not exceed B_{jk}, acquisition takes place.

In each plane j and at each target range 100k there are two limiting angles between which acquisition will occur with probability one and outside of which there will be no acquisition. Looking from the transducer toward the potential target define the set of limiting angles β_{jmk} with m = 1 for the limiting angle to port and m = 2 for the limiting angle to starboard. As shown in Figure 2, the reference axis for polar coordinates in the j th plane is the projection of the transducer axis at the time of ping transmission in that plane. Positive angles are measured clockwise from the reference axis in all planes j and in any horizontal plane. The pattern may not be centered on the axis and thus β_{jmk} may be positive or negative. Design or operational considerations will make it necessary to prohibit reception of signals from off the transducer axis in the azimuth plane in excess of some limiting angle α in the direction opposite to the search turn and α' in the direction of the search turn. This restricts the angles β_{jmk} as follows

$$\left| \beta_{j1k} - \phi_k \right| \leq \alpha$$

$$\left| \beta_{j2k} - \phi_k \right| \leq \alpha'$$

Recall that in plane j the maximum range for a non-negative B_{jk} was computed to be $100K'_j$ yards. For a straight running weapon this would be the maximum acquisition range in plane j. Actually an acquisition range of $100K'_j$ yards will not be realized in plane j in this problem because the weapon is turning at some rate. The energy

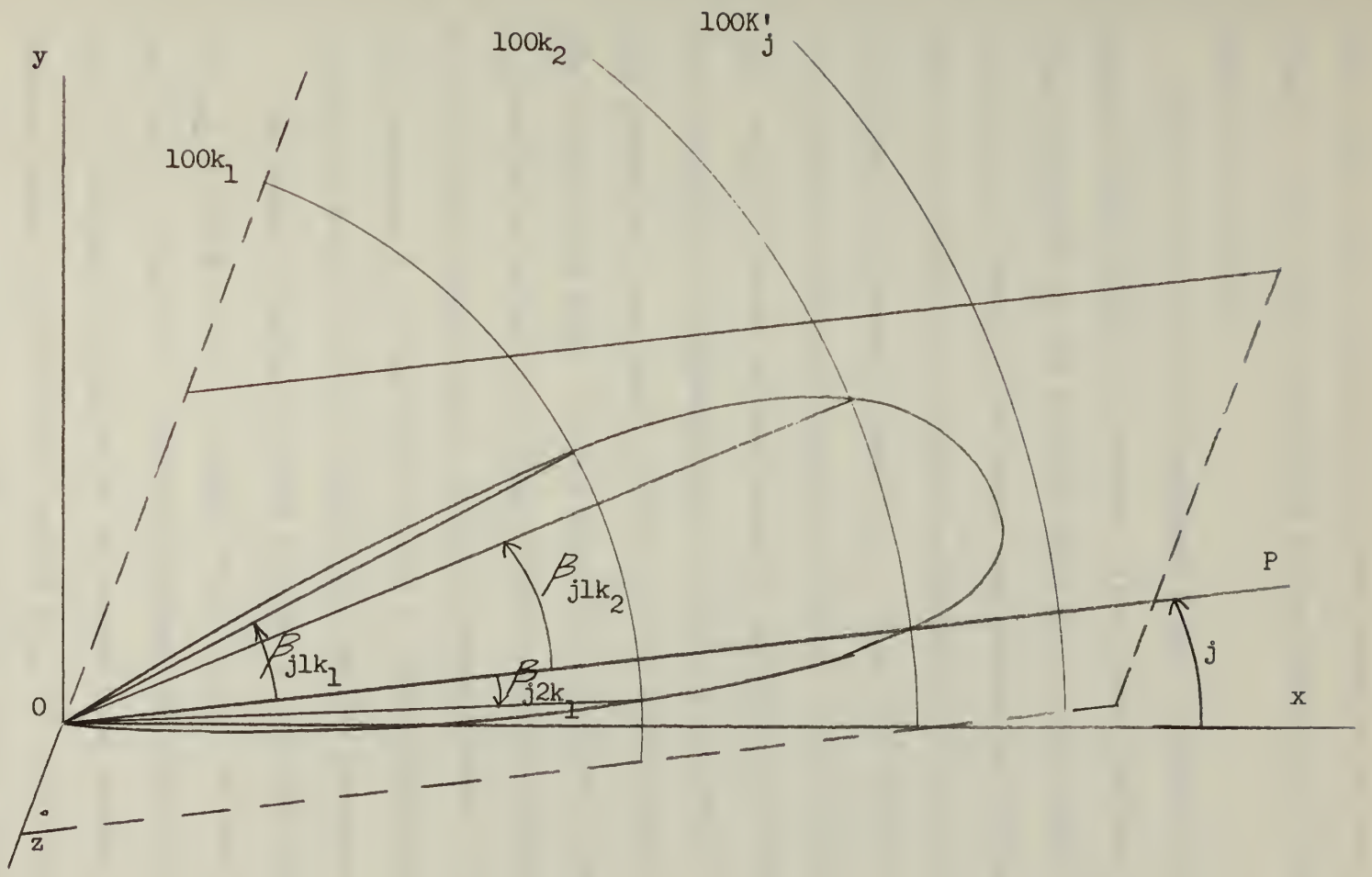


Figure 2: Acquisition limiting angles in the plane j. Line OP is the reference axis. The x axis is the transducer axis projection. In this case β_{j2k_2} is zero.

from a given source direction will never return as an echo in the same direction. Thus there will always be some penalty for off-axis transmission or reception.

Unless the turn rate is extremely high, the insonified volume of adjacent weapon pings will overlap. If a given ping scans part of the volume insonified by the previous ping, the overlap cannot be credited to the volume scanned by the later ping because the overlap volume was removed from the volume of target uncertainty by the previous ping. The concept of increased probability of detection because of overlap is not valid in this case because of the use of the definite range law. Thus all overlap volume should be credited to the ping which first scanned it. This would give all pings subsequent to the first ping during the first search turn an equal scanned volume less than that of the first ping. In this paper the situation is approximated by dividing the volume of overlap between the pings concerned to give all first turn pings an equal volume of insonification. This approximation decreases P_A (1) but the error in P_A (I) decreases with increasing I until the end of the first search turn when it becomes negligible.

The plane of maximum overlap is plane 0. In this plane, for small k, β_{02k} for ping i will be larger than $\delta + \beta_{01k}$ for ping (i+1). See Figure 3. Here δ is the angle between the axes of pings i and (i+1) or the angle through which the weapon turns between pings. As k becomes larger the differences between β_{02k} for ping i and $\delta + \beta_{01k}$ for ping (i+1) become smaller. Define β' to be the angle at which the two are equal. Thus the right-most angle for acquisition in any plane

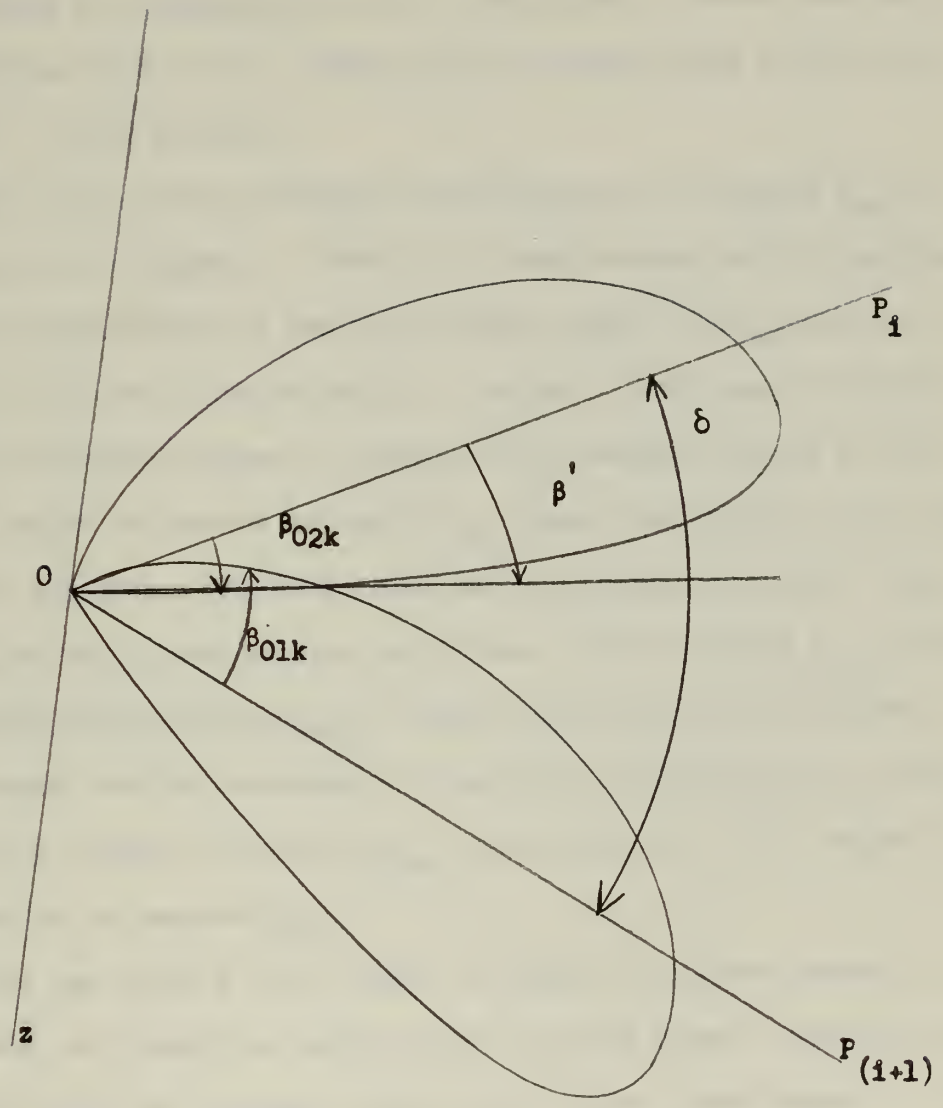


Figure 3: Overlap of insonified volume between adjacent pings.

is not greater than β' and the leftmost angle is not less than $\beta' - \delta$. Some of the extreme planes j may not be useful for acquisition at all. By including for consideration only those planes in which acquisition may take place the index j takes on the new range from J' below the axis to J'' above the axis.

The range or radial distance associated with the angle β_{jmk} is 100k yards. See Figure 4. There is a range maximum called the ping interval range defined by the time between pings ($1/PRR_s$) and the velocity of sound in the medium (c). Further, for planes inclined above the horizontal there is another range maximum defined by the range at which the radial at angle β_{jmk} breaks the surface of the water. Similarly for planes inclined below the horizontal there is a range maximum defined by the range at which the radial at angle β_{jmk} breaks the maximum target depth (D_{TM}). Volume scanned below the maximum target depth does not contribute to acquisition probability. Thus the acquisition range at any angle β_{jmk} is the minimum of the ranges just discussed to be denoted R_{jmk}.

Since the index k is no longer of value to indicate range it is useful now to relabel the radials such that the index k indicates the separation from the leftmost radial in the plane. See Figure 5. The leftmost radial is labeled (R_{j1}, β_{j1}). The general radial is (R_{jk}, β_{jk}). The rightmost radial is (R_{jK_j}, β_{jK_j}). For planes close to the axis, probably

$$\beta_{jK_j} = \beta' \quad \text{and} \quad \beta_{jK_j} - \beta_{j1} = \delta$$

This may not be true of the extreme planes.

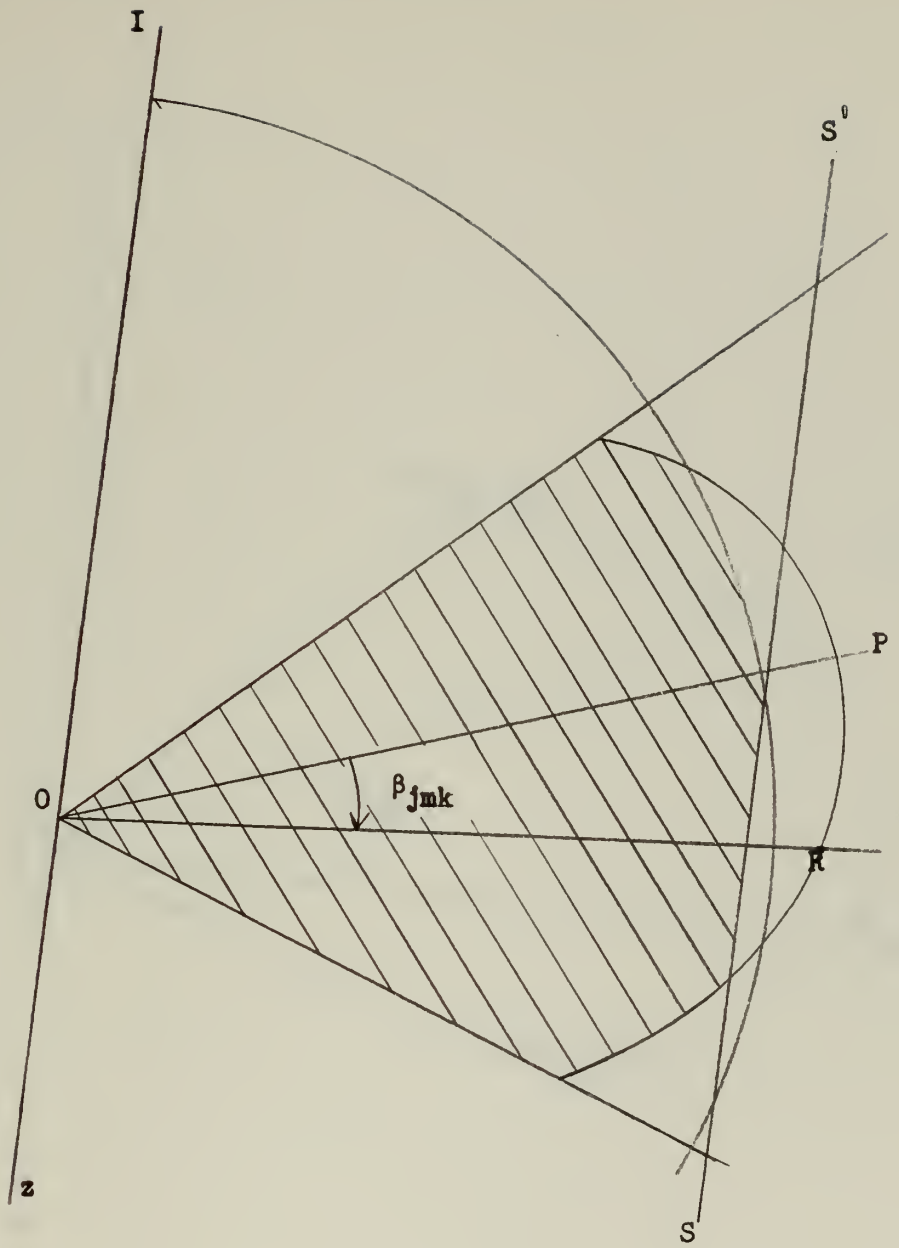


Figure 4: Acquisition range at angle β_{jmk}. Line OR has length 100k. Line SS' denotes the surface. Length OI is the ping interval range. Lined area is scanned.

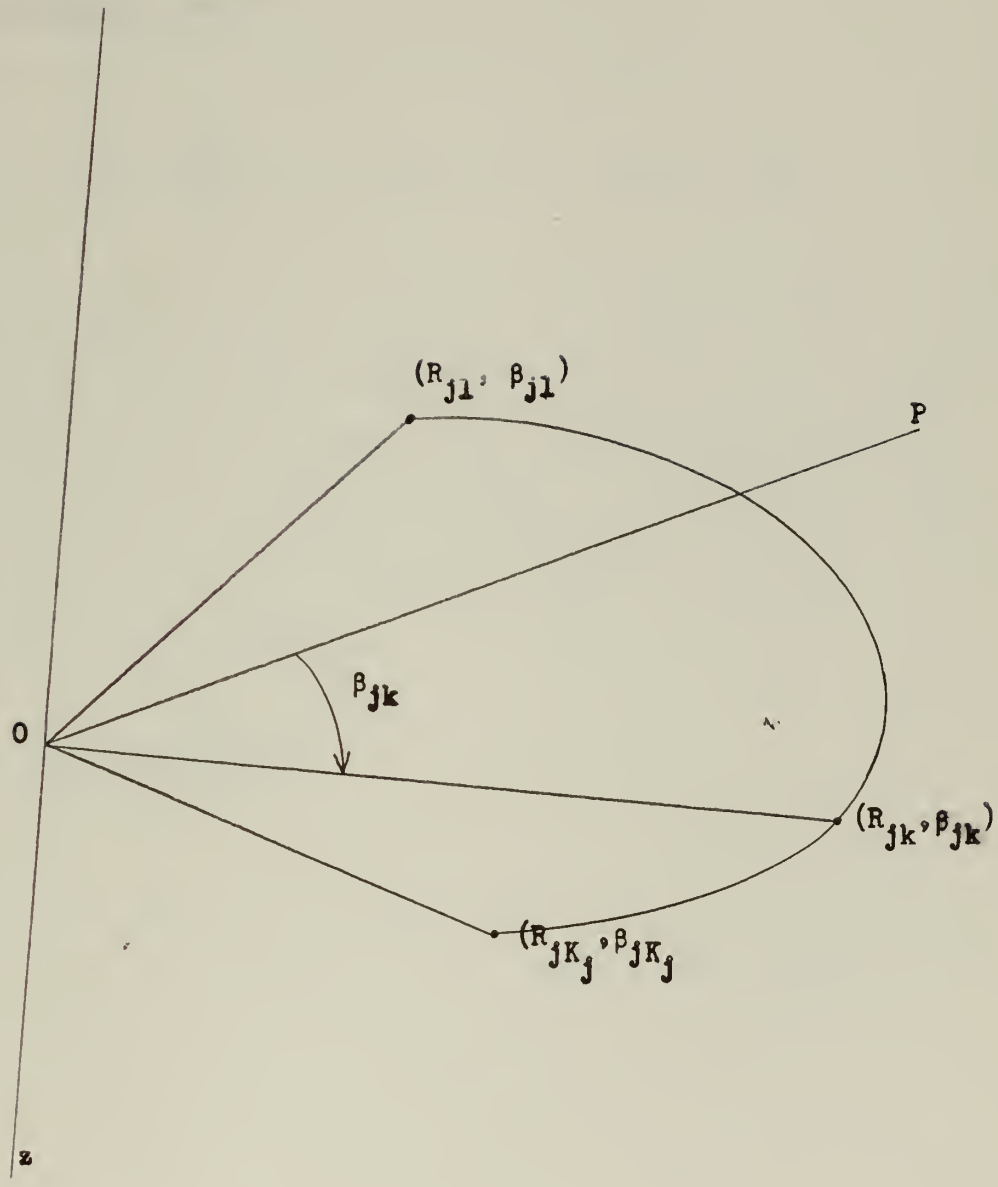


Figure 5: Relabeled radials in plane j.

The output of this step is tables of

β_{jk} and R_{jk}; $j = J', \cdots, J''$; $k = 1, \cdots, K_j$

plus the angles β' and δ .

3. Scanned Volume: General Case

The radials computed in step 2 define areas of insonification in each of a set of planes j. The area in each plane may be computed as the sum of a set of triangles as shown in Figure 6.

$$A_j = (1/2) \sum_{k=1}^{K_j-1} R_{jk} R_{j(k+1)} \sin (\beta_{j(k+1)} - \beta_{jk})$$

The irregular shape of the areas in the planes j makes it difficult to work with them in their present form. Define a set of isosceles triangles (one for each plane j) such that the triangle areas are equal to the scanned areas A_j in planes j and the common apex angle is the angle turned by the weapon per ping (δ). See Figure 7. The altitude of each triangle is

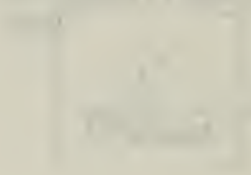
$$H_j = \left[\frac{A_j}{\tan(\delta/2)} \right]^{1/2}$$

The set of triangles so defined approximates the areas scanned and makes it possible to obtain the total scanned volume SV_{01} on ping zero on the first turn. Ping zero is a ficticious ping denoting the general case. The computation of SV_{i1} (scanned volume on i th ping of first search turn) is identical to that for SV_{01} to be developed next.

Define: μ = weapon angle of attack (pitch) (deg).

The plane j makes an angle of $(j+\mu)$ degrees with the horizontal.

The set of isosceles triangles is used to define a set of pyramids. A vertical cross-section of the set of pyramids is shown in Figure 8



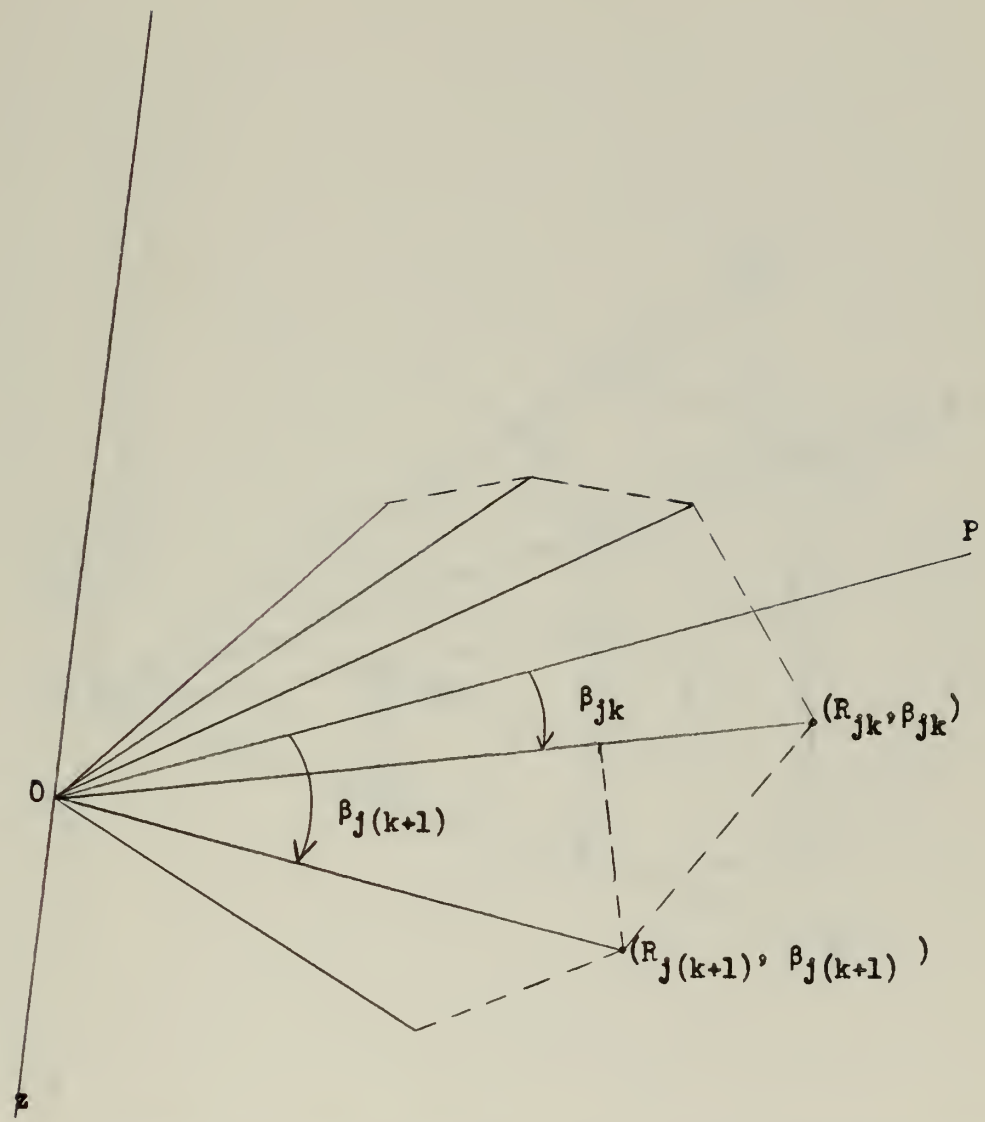


Figure 6: Area of insonification in plane j.

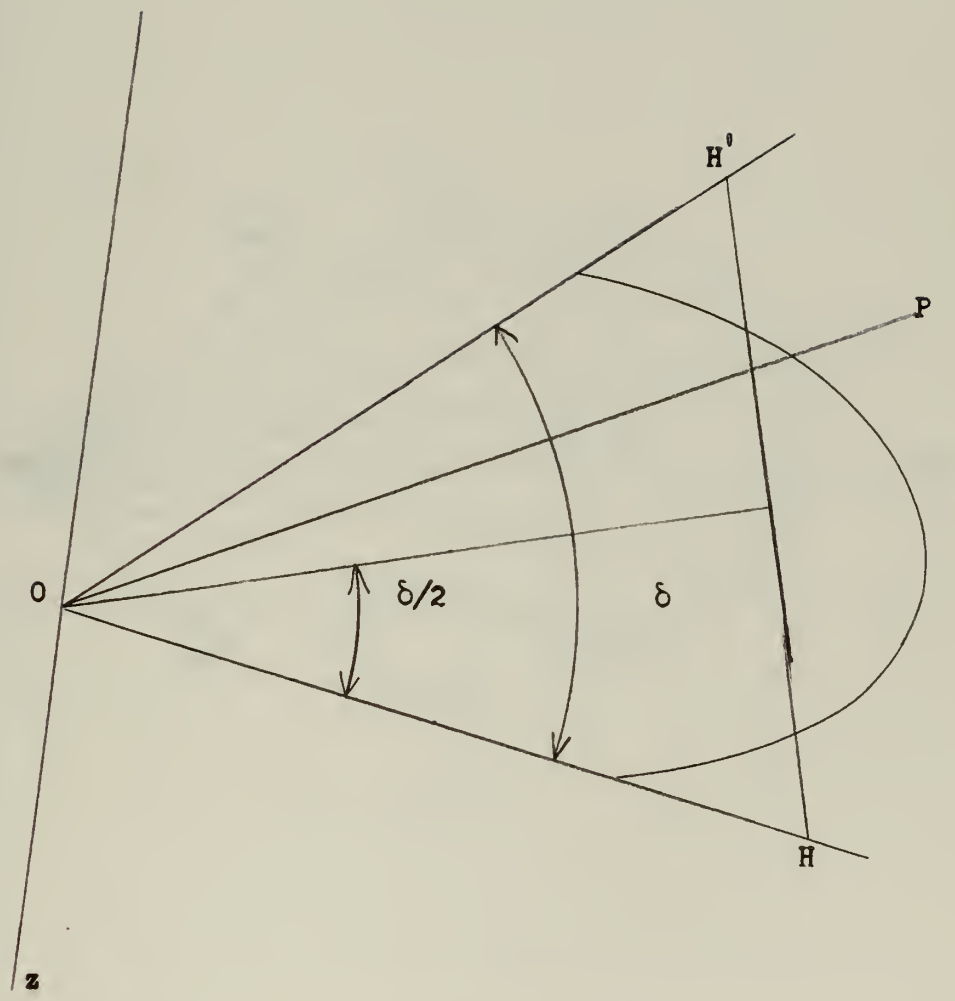


Figure 7: Equivalence of area A_j and isosceles triangle OHH' of altitude H_j and apex angle δ .

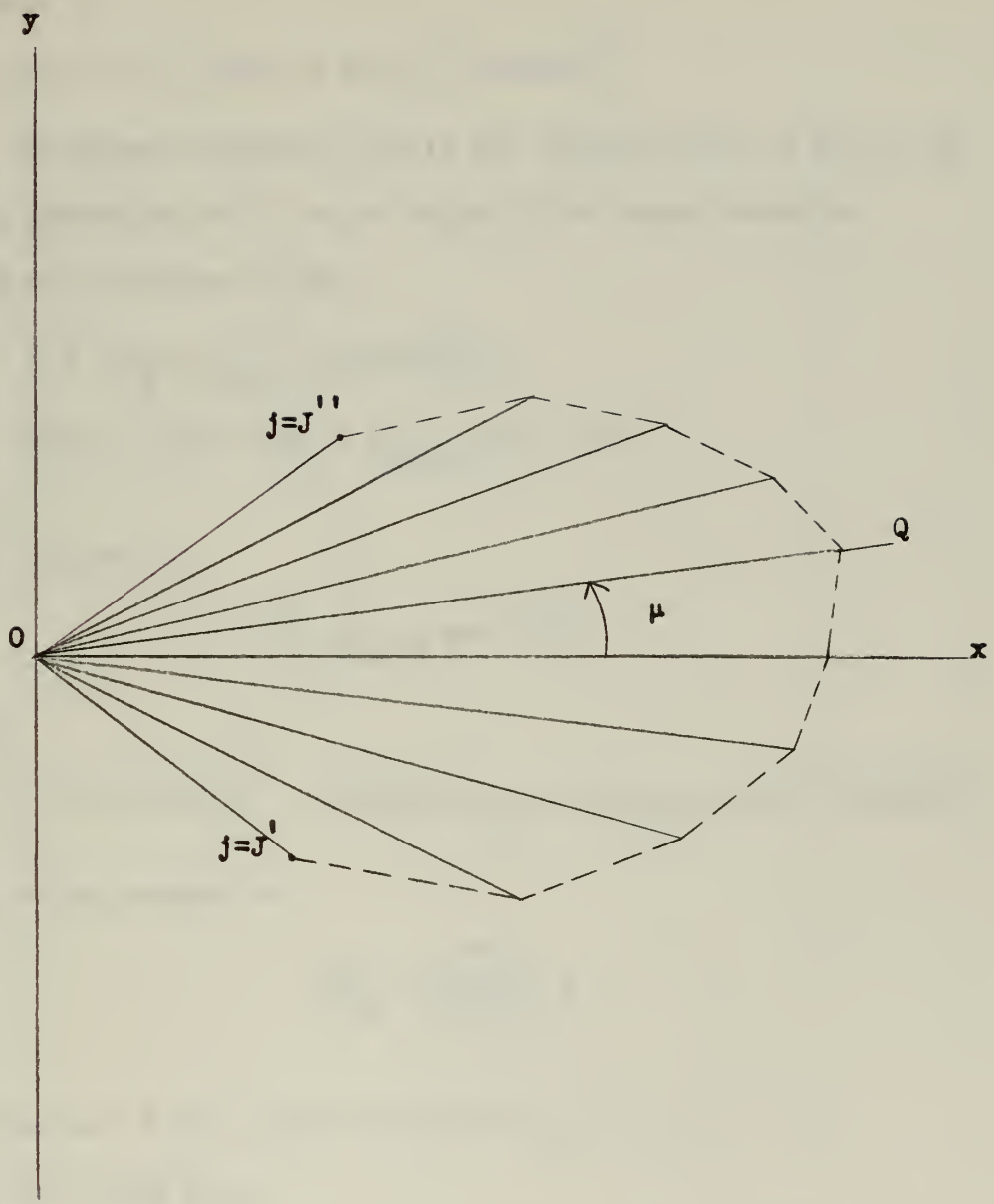


Figure 8: Vertical cross-section of insonified volume on ping zero. x is the horizontal axis, y is the vertical axis, and OQ is the transducer axis extension.

and a single pyramid is shown in Figure 9. The volume of a single pyramid is

$$v_j = 1/3 \quad \left[\text{area of base}\right] \left[\text{altitude}\right]$$

As shown in Figure 9, 2X is the average value of one of the base dimensions and Y is the value of the other dimension. Thus the base area is 2XY.

$$X = \left[(H_j + H_{(j+1)}) \tan(\delta/2)\right]/2$$

Since $\sin\phi = h/H_j = (H_{(j+1)} \sin 1^0)/Y$

the altitude is

$$h = (H_j \, H_{(j+1)} \sin 1^0)/Y$$

and

$$V_j = (1/3)(2XY)h = (1/3)(H_j+H_{(j+1)}) \, H_j \, H_{(j+1)} \sin 1^0 \tan(\delta/2)$$

The volume scanned is

$$SV_{01} = \sum_{j=J'}^{J''-1} V_j$$

The output of this step is tables of H_j; $j = J', \cdots, J''$ and the volume SV_{01}.

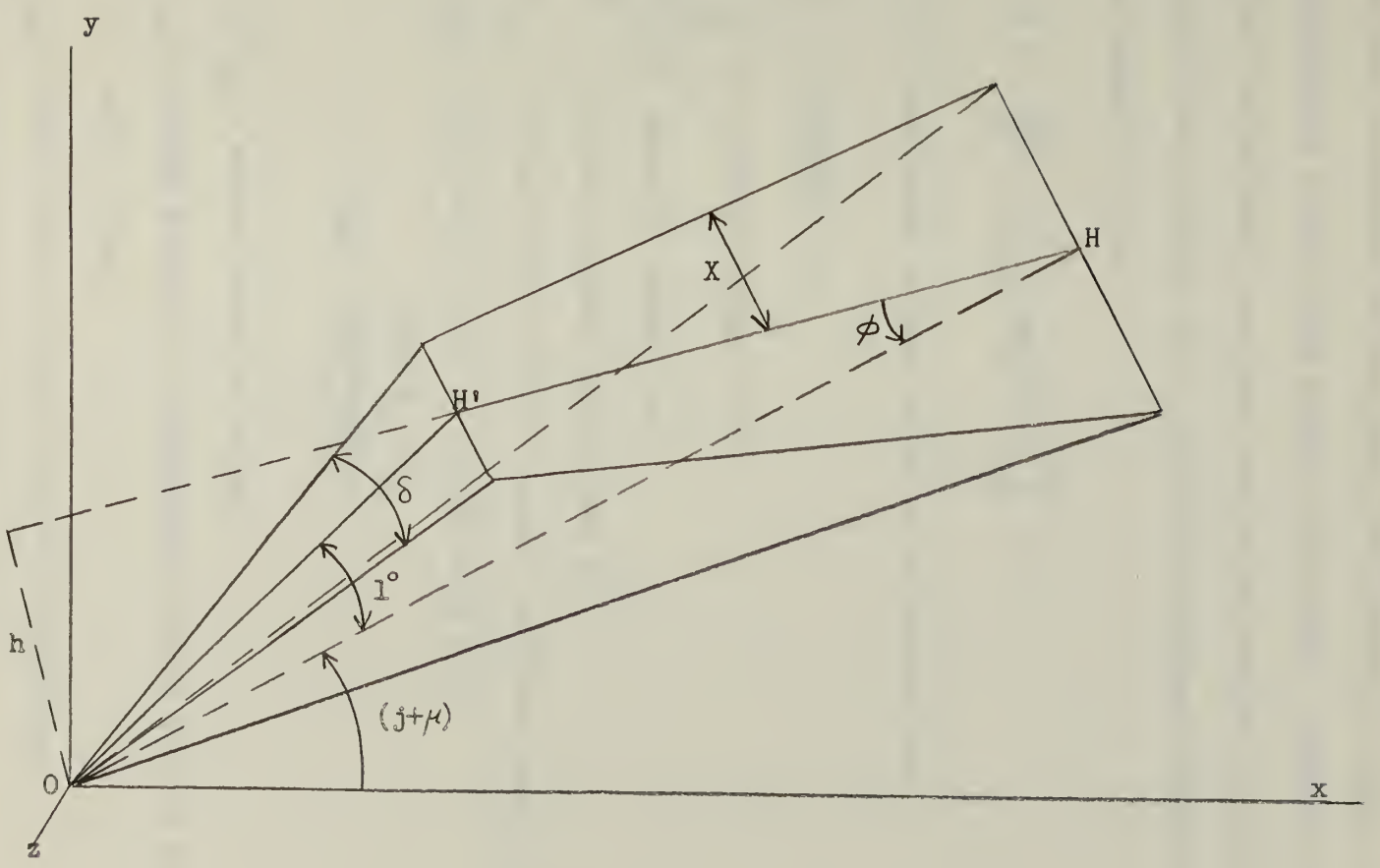


Figure 9: Pyramid volume V_j. Line OH is H_j. Line OH' is $H_{(j+1)}$. Line HH' is Y.

4. Special Volume Elements: General Case

For the case of search at constant depth in a continuous circle as is being considered in this paper, the situation changes drastically upon completion of the first search turn circle. Subsequent search turns will effectively scan much less volume because the weapon retraces its path of the previous turn. Define the maximum available number of search turns (U) to be

$$U = \left[\left[(\bar{W}' - w_{OSD})\ PRR_s / M\ w_{1t}\right]\right] + 1$$

where M is the number of ping cycles in one search turn

$$M = ||360/\delta|| + 1$$

$[[x]] \equiv$ greatest integer $\leq x$

$||x|| \equiv$ greatest integer $< x$

$\bar{W}'$ = maximum allowable weapon fuel (lb)

w_{1t} = fuel expenditure rate at search depth and speed (lb/sec)

w_{OSD} = fuel used in dive to search depth (lb)

$$w_{OSD} = \frac{(1/3)\ D_{WS}\ w_{Ot}}{\sin\phi_{DS}\ S_{WD}}$$

$1/3\ D_{WS}$ = weapon search depth (yd)

w_{Ot} = average weapon fuel expenditure rate in dive to search depth (lb/sec)

S_{WD} = weapon speed in dive to search depth (yd/sec)

ϕ_{DS} = angle of dive to search depth (deg)

Consider a side view or vertical cross-section of the scanned volume of a single ping as in Figure 10. The vertically lined area is that scanned by the weapon on the i th ping of the first search turn. Recall assumption 6 that the target opens radially from the weapon position and assumption 12 about weapon orientations on different search turns. Define a distance of TES yards as the distance traveled by the target during the time it takes the weapon to complete one search turn. The weapon scanned area on the i th ping of the second search turn is equivalent to that of the areas lined sloping downward to the right on Figure 10. The weapon actually scans the same geographic volume on each search turn but since the target is assumed to open radially the situation is equivalent to translating the area scanned on the first turn a distance TES to the left for each subsequent turn. If the target is in the area lined sloping downward to the left on the i th ping of the first search turn, he will 'escape' prior to the i th ping of the second search turn. Similarly for subsequent search turns. A target above the critical depth CDA or below the critical depth CDB may escape but a target at any intermediate depth will be acquired eventually if fuel supply permits because of the assumption of an opening target. The search turns after the first are desirable because of the possibility of acquiring a target 'trapped' inside the insonified volume of the first search turn. Treatment of scanned volumes on the second and subsequent turns will differ depending on whether the volume is above or below the depths CDA and CDB. Although the areas scanned above depth CDA on search turns 1 and 2 are equal in

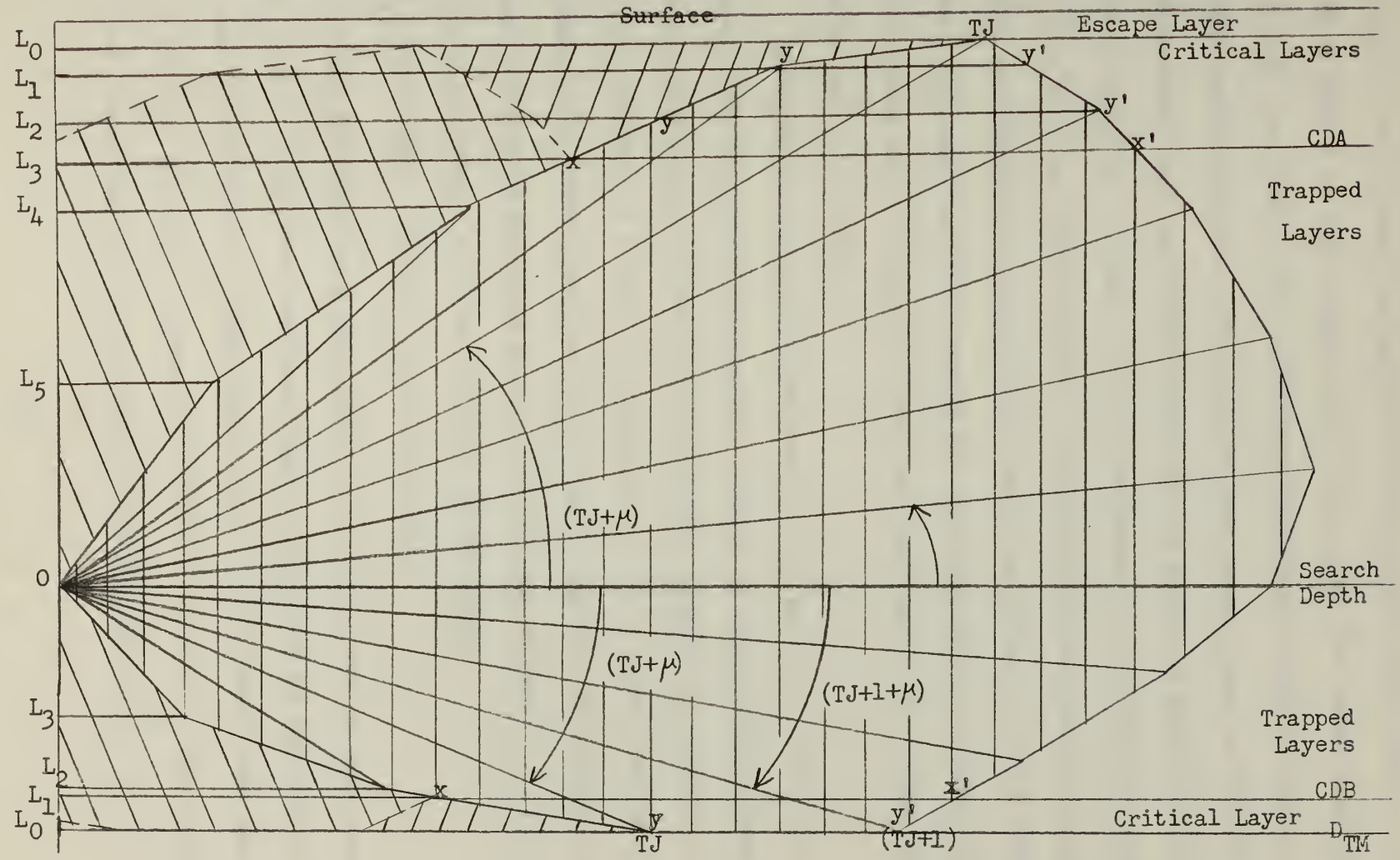


Figure 10: Vertical cross-section of a single ping. Lines xx' have length TES. Lines yy' and lines xx' have length L_d.

Figure 10, the volumes represented by these areas are not equal. A top view of ping i is a pie shaped slice which makes the volume associated with the first search turn area larger than that of the corresponding second search turn area. Thus each volume element must be calculated independently and the volume scanned by the weapon on ping i of search turn u (SV_{iu}) is the sum of several independently calculated volumes. In this step the volume elements in the general case (i=0) are computed. These are modified as necessary for each ping orientation in a later step. Although Figures 11 and 12 and the development to follow involve the volumes above the search depth, the development for volumes below the search depth is identical.

Consider the vertical cross-section of Figure 10. The cross-section has been split into several regions or volume layers identified on the left margin by their bottom boundary labeled L_d such that the bottom boundary of a level (or layer) at or above depth CDA has a length of L_d inside the first turn scanned region. The planes j are considered using the plane (j = TJ) of maximum $H_j \left|\sin (j + \mu)\right|$ as a reference plane. In Figure 10 below search depth

$H_{TJ} \left|\sin (TJ + \mu)\right| = H_{(TJ+1)} \left|\sin (TJ+1+\mu)\right|$.

In this case where more than one of the planes j has the same maximum $H_j \left|\sin (j + \mu)\right|$, the reference plane is chosen to be that with the least $H_j \cos (j + \mu)$. Above search depth, each plane j such that $H_j \sin (j + \mu) < D_{WS}$ - CDA and $j > TL$ defines a level L_d. Also each plane j such that $H_j \sin (j+ \mu) \geq D_{WS}$ - CDA defines a level L_d.

Referring to Figure 11, the volume elements $LADV_{du}$, $MADV_{du}$, and $RADV_{du}$ ($d = 1, \cdots, \bar{D}$; $u = 2, \cdots, U$) are significant. The subscript d indicates the level L_d and the subscript u indicates the search turn number. The level $L_{\bar{D}}$ is the final level above the critical depth CDA and $L_{\bar{D}}$ = TES. The corresponding volumes below the search depth are $LBDV_{du}$, $MBDV_{du}$, and $RBDV_{du}$ ($d = 1, \cdots, \underline{D}$; $u = 2, \cdots, U$). The scanned volume in level L_d above the search depth on the u th search turn is

$$ASV_{du} = LADV_{du} + MADV_{du} + RADV_{du}.$$

One or more of these volume increments may be zero as is $LADV_{d3}$ in Figure 11. The corresponding scanned volume below the search depth is BSV_{du}.

Referring to Figure 12,

the volumes AMV_{du}; $d = \tilde{\bar{D}}+1; \cdots, \tilde{\bar{D}}'$; $u = 1, \cdots, U$ are significant. The corresponding volumes below the search depth are denoted

BMV_{du} ($d = \underline{D} + 1, \cdots, \underline{D}'$; $u = 1, \cdots, U$)

For levels L_d below depth CDA and above search depth and for the u th search turn, AMV_{du} is the trapped volume of target uncertainty remaining after the ping cycle is complete. The volume scanned in level L_d above the search depth on the u th search turn is thus

$$ASV_{du} = AMV_{d(u-1)} - AMV_{du}$$

The corresponding scanned volume below the search depth is BSV_{du}.

In this step all calculations are done for ping zero and the quantities computed for ping zero are saved to be modified later as appropriate for the i th ping. The total scanned volume for ping zero of the u th search turn is

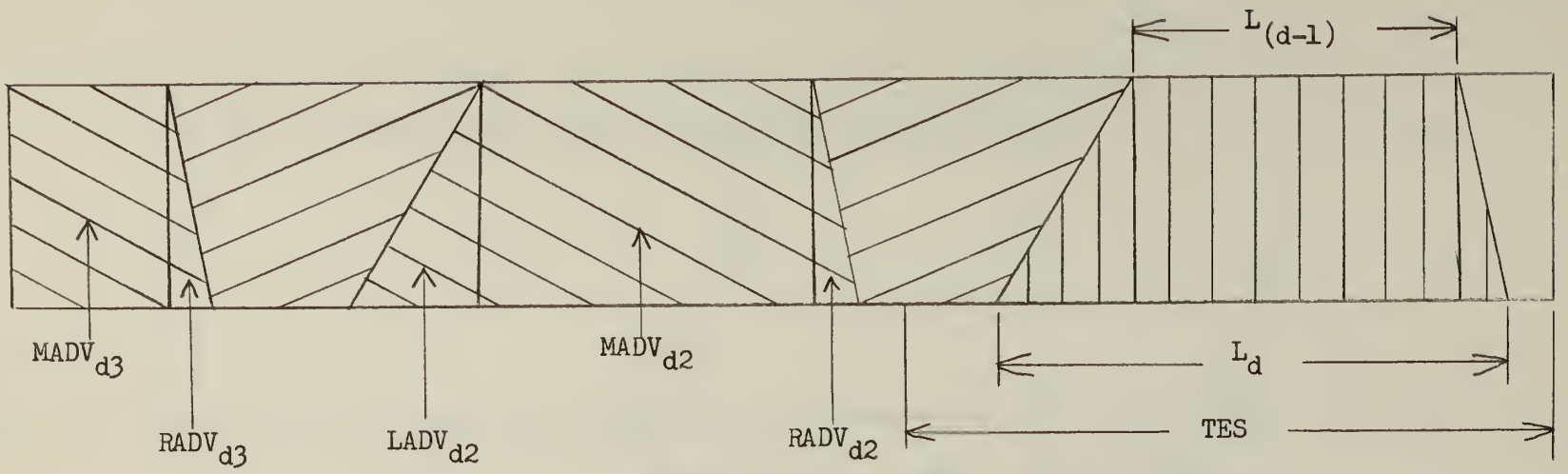


Figure 11: Cross-section of sample level L_d above critical depth CDA. Area actually scanned is lined vertically. Equivalent areas scanned have lines sloping down to the right. Areas of target escape have lines sloping down to the left.

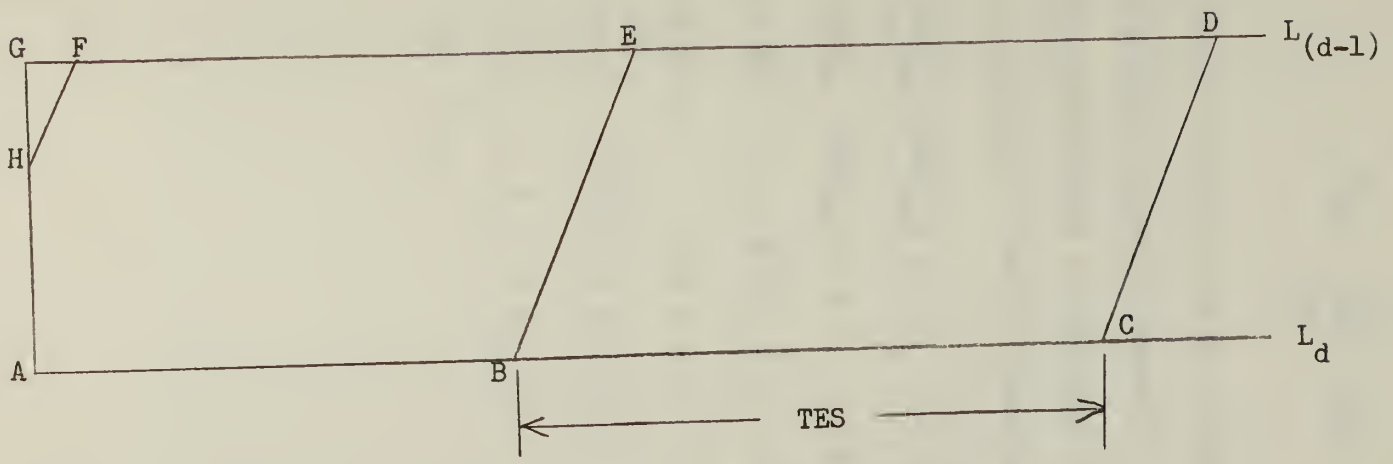


Figure 12: Cross-section of sample level L_d below critical depth CDA. Area ACDG represents volume AMV_{d1}. Area ABEG represents volume AMV_{d2}. Area HFG represents volume AMV_{d3}. Area to right of line CD represents actual scanned volume.

$$SV_{Ou} = \sum_{d=1}^{\overline{D}'} ASV_{du} + \sum_{d=1}^{\underline{D}'} BSV_{du}$$

The output of this step is the number of pings (M) in a search turn, the maximum available number of search turns U, several tables of parameters described in appendix C4 used in computing the special volumes, and tables of the following special volumes for ping zero:

$LADV_{du}$, $MADV_{du}$, $RADV_{du}$; $d = 1, \cdots, \overline{D}$; $u = 2, \cdots, U$

$LBDV_{du}$, $MBDV_{du}$, $RBDV_{du}$; $d = 1, \cdots, \underline{D}$; $u = 2, \cdots, U$

AMV_{du}; $d = \overline{D} + 1, \cdots, \overline{D}'$; $u = 1, \cdots, U$

BMV_{du}; $d = \underline{D} + 1, \cdots, \underline{D}'$; $u = 1, \cdots, U$

ASV_{du}; $d = 1, \cdots, \overline{D}'$; $u = 2, \cdots, U$

BSV_{du}; $d = 1, \cdots, \underline{D}'$; $u = 2, \cdots, U$

SV_{Ou}; $u = 2, \cdots, U$

5. First turn scanned volume: i th ping

Throughout the search phase, the weapon is displaced from the center of the target volume cylinder a horizontal distance R_o. The base radius of the target cylinder is some distance R_I at the time the weapon splashes into the water. The radius then increases at a rate (S_{TC}) equal to the target cruise speed to a distance R_1 at the time of the first weapon ping. Thereafter the radius increases at a rate (S_{TS}) equal to the alerted target escape speed and has a value R_i upon commencement of the i th ping cycle.

In the situation of Figure 13, a top view of the target cylinder of increasing volume with the first few ping orientations and scanned volumes in the first search turn, the insonified volume of ping 1 includes a large volume outside the target cylinder. Such volume does not contribute to acquisition probability because there is an assumed zero probability that the target is outside. Therefore the total insonified volume must be reduced to the useful scanned volume inside the target cylinder. In the situation depicted, the amount of discarded volume is reduced with each ping until ping 9 when all of the insonified volume is within the target volume cylinder.

The projection of the radial (R_{jk}, β_{jk}) in plane j to the horizontal plane through the transducer is the line OP of Figure 14 denoted

R'_{jk} where

$$R'_{jk} = R_{jk} \cos (j + \mu)$$

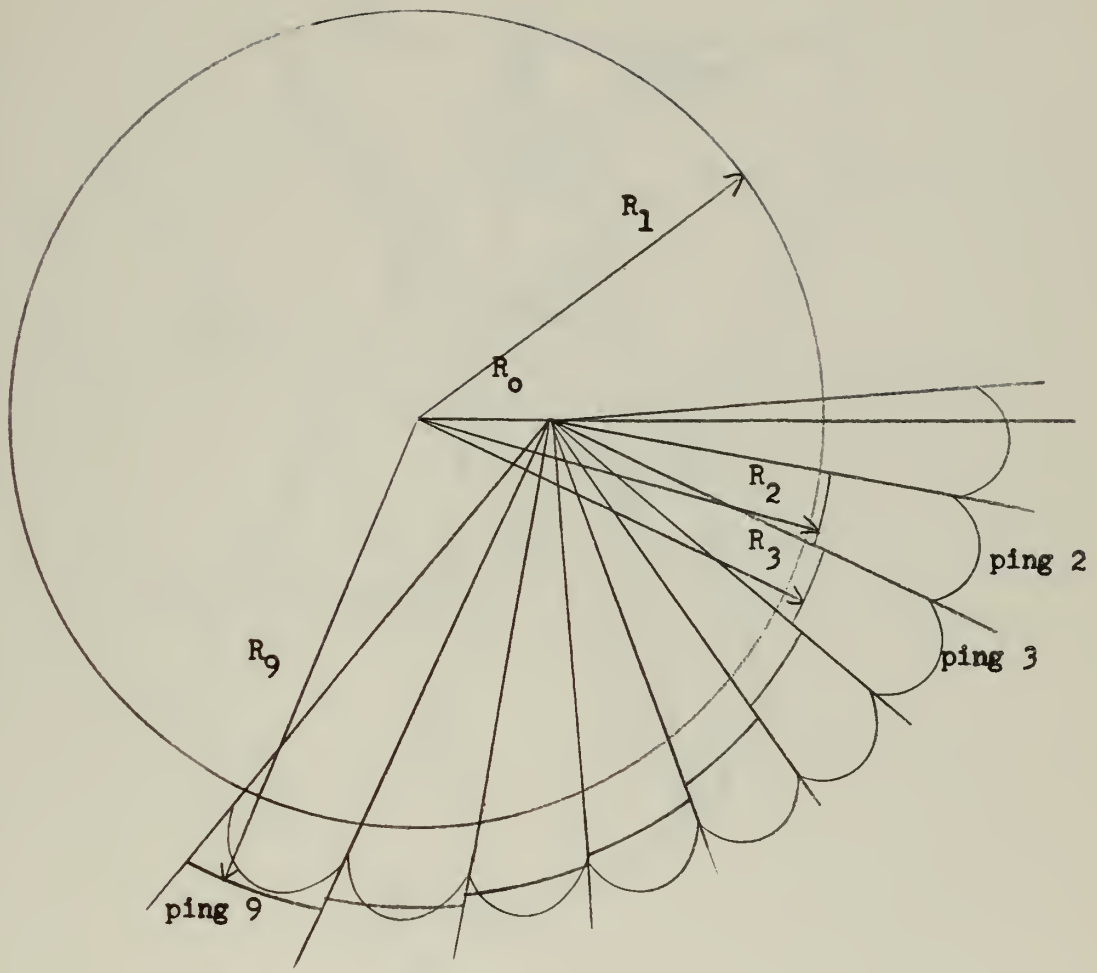


Figure 13: Top view of target cylinder of increasing volume with the insonified volumes of several sample pings of the first search turn depicted.

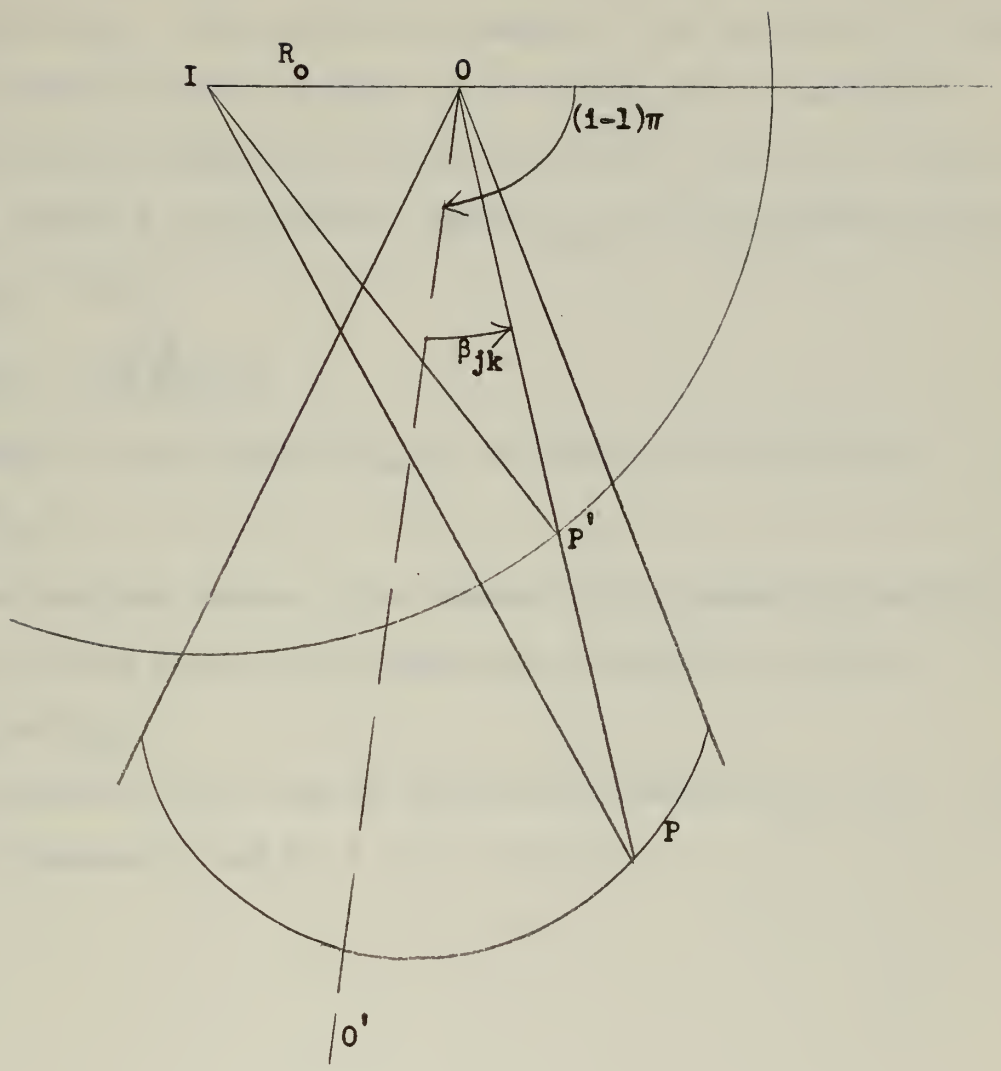


Figure 14: The projection of plane j in the horizontal at search depth. Line OO' is the plane j reference axis projection. Line IP is CR_{jk}. Line OP is R'_{jk}. Line IP' is R_i. Line OP' is R'_{ijk}.

Use of the law of cosines enables determination of the distance I P denoted CR_{jk}. This distance is compared to the radius (R_i) of the target cylinder on the i th ping. If the radius (R_i) is smaller, it defines the new triangle IOP' and the projection of the side OP' back to the plane j defines a new acquisition range (R_{ijk}) to be associated with the angle β_{jk}. That is

$$R_{ijk} = \frac{R'_{ijk}}{\cos(j + \mu)}$$

However if the distance CR_{jk} is the lesser of the two then

$R_{ijk} = R_{jk}$

The remaining portion of the computation for scanned volume (SV_{il}) on the i th ping of the first search turn is identical to that in step 3 for SV_{01}.

The output of this step is the table of volumes SV_{il}; i = 1, $\cdots$, M-1 and the distances R_o and R_i; i = 1, $\cdots$, M-1.

6. Scanned volumes subsequent to first turn: i th ping

Just as the first turn scanned volume had to be corrected for the i th ping because of the effect of the target cylinder radius, so must the scanned volumes on subsequent turns.

Define the intersection of the ping i bisector with the target cylinder radius at transmission of ping i to be a distance CH from the weapon.

Referring to the vertical cross-section of the insonified volume in Figure 15, verticals through four of the possible intersections have been plotted and labeled CH1 through CH4. The effect of each of these four cases will be discussed as it applies to a level above and a level below the critical depth CDA (i.e: levels 2 and 4). The treatment of the situation below the search depth is identical.

In the case of CH1 the first turn scanned volume is reduced. However the scanned volumes for all subsequent turns are the same as those computed in step four for the general ping. This is because CH1 lies to the right of all areas representing equivalent volumes.

In the case of CH2 there is still no reduction for volumes in level 2. However there is a small reduction in the volume scanned on the second search turn in level 4. The volume reduction is that volume associated with the area in level 4 to the right of CH3.

It is computed as a reduction in AMV_{41} (the volume missed in level L_4 above search depth on the first search turn).

Thus $(ASV_{22})_{\text{ping i}}$ equals $(ASV_{22})_{\text{ping 0}}$ but

$(ASV_{42})_{\text{ping i}}$ is less than $(ASV_{42})_{\text{ping 0}}$.

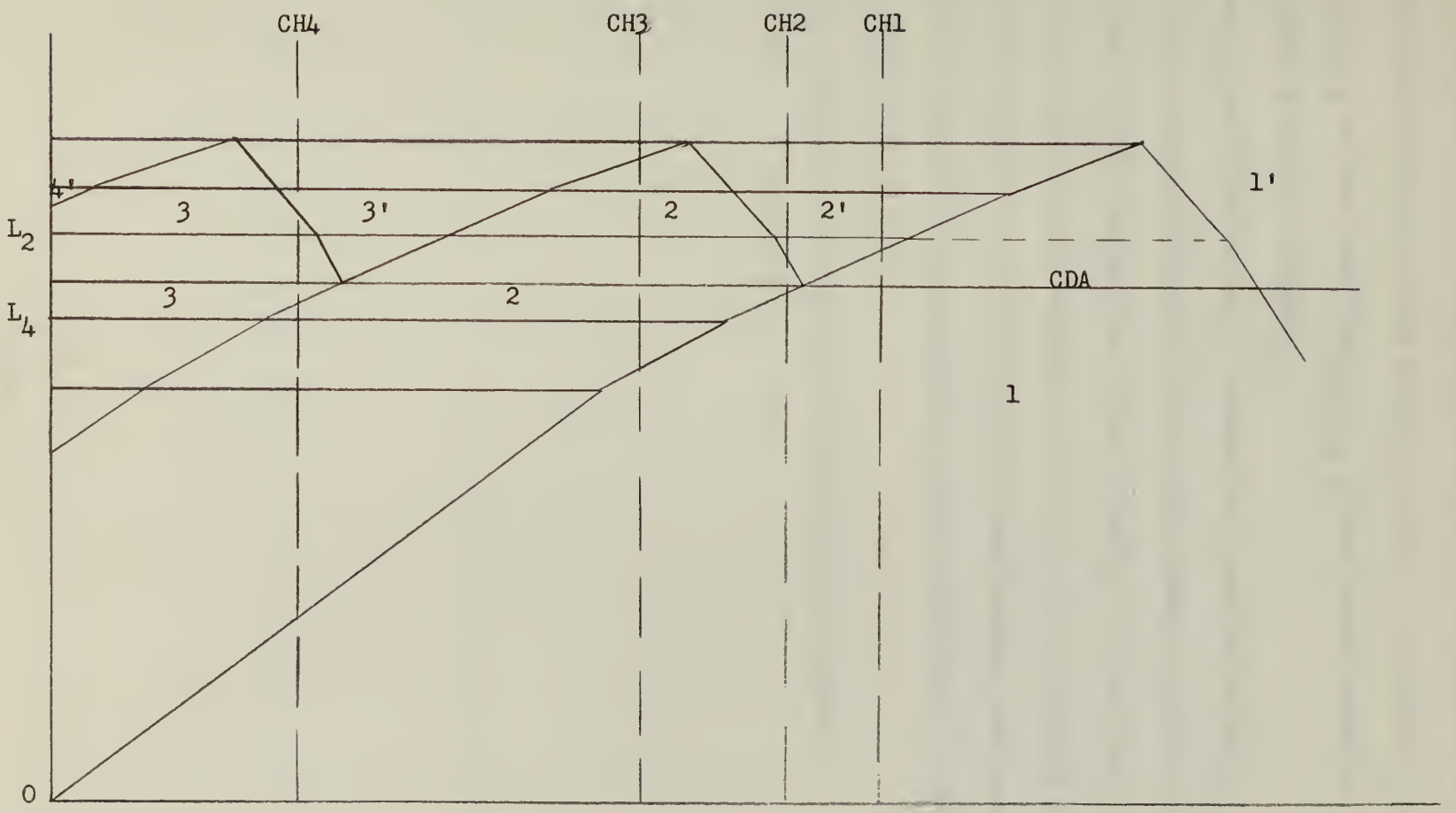


Figure 15: Vertical cross-section of ping insonified volume. Volumes are scanned on the search turn as numbered. Escape volumes have primed numbers according to the search turn of escape.

In the case of CH3 there is a reduction in the scanned volume on the second search turn in both levels L_2 and L_4. In level L_2 the volume $(ASV_{22})_{ping\ i}$ is less than $(ASV_{22})_{ping\ 0}$ by an amount equal to $RADV_{22}$ plus a portion of $MADV_{22}$.
In level L_4 the reduction still amounts to a portion of AMV_{41}.

In the case of CH4 there is no scanned volume in level L_2 above search depth on the second turn. That is $ASV_{22} = 0$ for ping i. Further there is a small reduction in level L_2 in the scanned volume for the third search turn. For level L_4 there remains some scanned volume on the second turn and there is also a reduction in scanned volume for the third turn.

The output of this step is a table of scanned volumes SV_{iu}; i = 1, ···, M-1; u = 2, ···, U.

7. Target Volume: i th ping

Consider that portion of the target cylinder volume computed at a base radius of R_1 within the radials bounding the ith ping. See Figure 16. The area contained between the boundary radials of the i th ping and within the cylinder of radius R_1 multiplied by the height of the cylinder is this volume. The probability that the target is in this i th volume slice of the target cylinder is the volume of the slice divided by the total cylinder volume. This probability (PQ_i; i = 1, ···, M-1) remains constant throughout the problem because of the opening target assumption number six.

From Figure 16:

$$PQ_i = \frac{\text{area of horizontally lined region}}{\pi R_1^2}$$

The target volume (TV_i; i = 1, ···, M-1) associated with the i th ping on the first search turn lies between the same radial limits but within a target cylinder of radius R_i instead of R_1. That is

$$TV_i = (\text{area of vertically lined region})\ D_{TM}$$

Figure 16 shows the angle (δ') assigned to the M th ping to be less than δ. It is

$$\delta' = 360 - (M-1)\delta$$

This smaller angle is required to complete the search turn so that the i th ping of the u th turn is oriented in the same direction as the i th ping in the (u + 1) th turn. Thus the M th ping is in part ficticious.

The output of this step is thus the tables of TV_i; i = 1, ···, M-1 and of PQ_i; i = 1, ···, M-1 and the angle δ'.

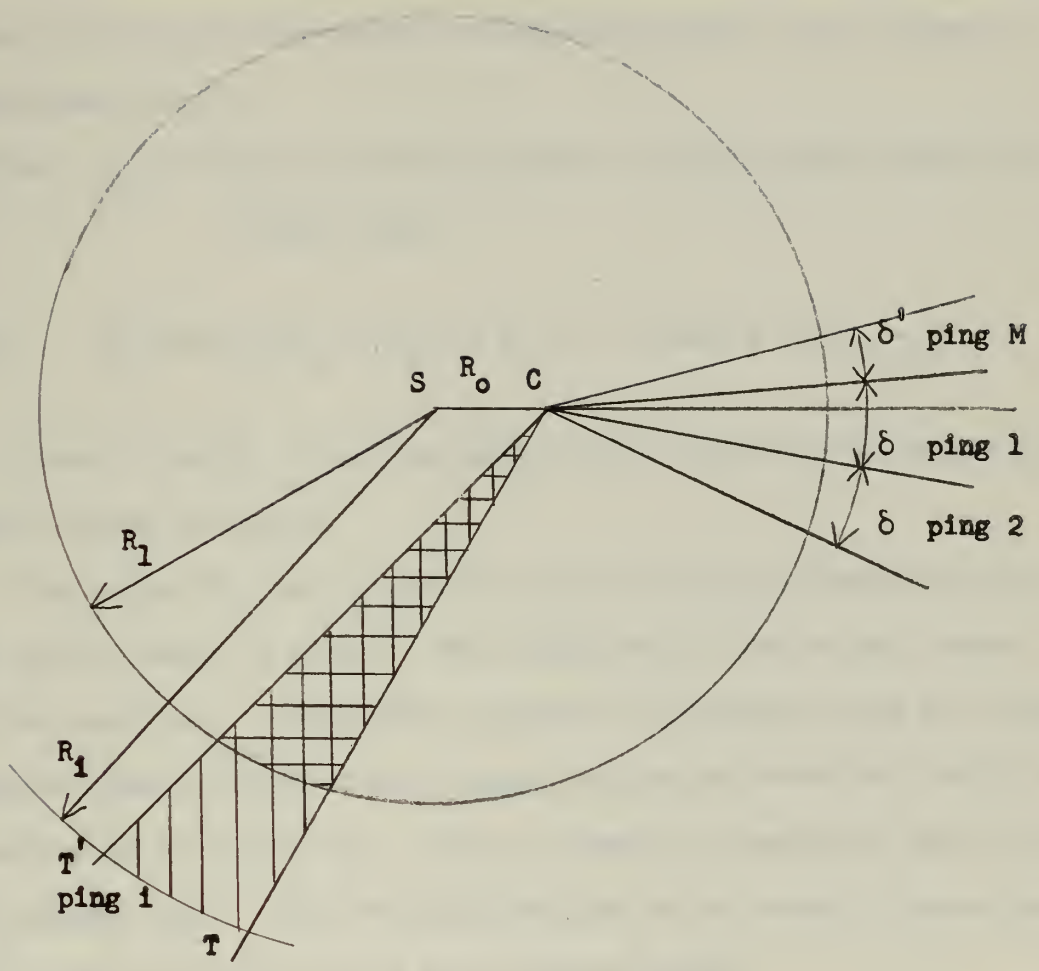


Figure 16: Top view of target cylinder of increasing volume. Lines CT and CT' are boundary radials of ping i. Horizontally lined area is PQ_i times πR_1^2. Vertically lined area times cylinder height is TV_i.

8. Acquisition Probabilities: I pings

The instantaneous probability of acquisition on the i th ping is simply the ratio of the volume effectively scanned to the volume of target uncertainty.

Define: g_{iu} = P [acquisition during the i th ping cycle of the u th search turn]

Then $$g_{iu} = SV_{iu}\ PQ_i \ / \ TV_i \ ; \ i = 1, \cdots, M-1; \ u = 1, \cdots, U$$

Recall that PQ_i is the probability that the target is within the radial limits of ping i.

The volume TV_i is the total volume of target uncertainty within the radial limits of ping i. The volume SV_{i1} is the volume scanned on the first turn. The volumes scanned on subsequent turns are volumes measured inside the first turn scanned volume and under the conditions existing for the first turn. These volumes are then moved intact with the opening taget. Thus the applicability to all turns of the volume TV_i measured using the first turn cylinder radius.

The instantaneous probability of detection on the M th ping is approximated by

$$g_{Mu} = g_{(M-1)u}\ (\delta'/\delta) + g_{1(u+1)}\ (\delta - \delta')/\delta$$

The M th ping is alloted the same time interval as any other ping. Errors involved should be small.

There remains the requirement to relabel the g_{iu} to be g_i; $i = 1, \cdots, I$, to compute I as a function of w_1 as described in the beginning of the chapter, to compute P_A as a function of I, and to tabulate the values of P_A versus w_1 as w_1 is allowed to vary from $\underline{W}'$ to $\bar{W}'$. This final table is the input to the dynamic programming model.

9. Input Requirements

A. Medium

c = velocity of sound in water (yd/sec)

a = attenuation coefficient (db/yd)

10 log m_s = surface reverberation scattering coefficient (db)

10 log m_V = volume reverberation scattering coefficient (db)

B. Target

TS = target strength (db)

TN = radiated noise level of the target (db at 1 yd)

D_{TM} = maximum target depth (ft)

S_{TS} = target speed during weapon search (yd/sec)

S_{TC} = target cruise speed (prior to becoming alerted on the first ping cycle) (yd/sec)

C. Weapon

WS = transmitted on-axis source level of weapon (db at 1 yd)

ΔT = ping length (sec)

PRR_s = weapon active sonar pulse repetition rate for search (pings/sec)

Δf = receiving bandwidth (cycles/sec)

WEL_k = weapon enabling level for a range of 100k yds (db)

SN = equivalent isotropic self-noise level at search speed, depth, and turn rate (db)

RD = recognition differential for 50% probability of acquisition (db)

b_1 = normalized pattern function in the vertical plane for transmitting pattern (a numeric)

b_2 = normalized pattern function in the vertical plane for receiving pattern (a numeric)

b_3 = normalized pattern function in the azimuth plane for transmitting pattern (a numeric)

b_4 = normalized pattern function in the azimuth plane for receiving pattern (a numeric)

J = limiting angle for off-axis reception in the vertical plane (deg)

α, α' = limiting angles for off-axis reception in the azimuth plane (deg)

DR = directivity index for receiving (db)

μ = angle of attack (pitch) of weapon (deg)

ω_S = search rate of turn (deg/sec)

ϕ_{DS} = angle of dive in diving to search depth (deg)

D_{WS} = weapon search depth (ft)

S_{WD} = weapon speed in dive to search depth (yd/sec)

S_{WS} = weapon speed during search

$\bar{W}'$ = maximum allowable weapon fuel weight (lb)

w_{lt} = weapon fuel expenditure rate at search depth and speed (lb/sec)

w_{Ot} = average weapon fuel expenditure rate in dive to search depth (lb/sec)

D. Delivery weapons system capability

R_I = radius of target volume cylinder at time of weapon splash

Statistical averages and distributions are available for the medium inputs. Intelligence reports and estimates or CNO requirements may provide the target inputs except for the target speeds which may be selected in the higher ranges in accordance with criterion one. The weapon inputs are available from weapon designers at least in the form of educated estimates. Finally the input R_I may be estimated from observations of the performance of available delivery weapons systems.

CHAPTER IV

PROBABILITY OF HIT

Consider a tactical situation in which a weapon acquires a submarine target at some initial slant range and make the following assumptions.

Assumption 1: The weapon makes an attack and a series of independent reattacks on the target culminating in either a hit or fuel exhaustion.

Assumption 2: At the time of acquisition, the target is opening at maximum speed. Target course and speed are constant throughout the attack and reattack.

Assumption 3: On the attack and after reacquisition on each reattack the weapon maintains contact and closes at attack (maximum) speed.

Assumption 4: The target operates in depth within specified minimum and maximum limits such that the maximum depth limit is not greater than the weapon maximum operating depth.

Assumption 5: The depth at which a hit may take place is a random variable denoted the hit depth which has an assumed probability density function.

Assumption 6: The average weapon depth for the attack is the arithmetic average of the search depth and the average hit depth. The weapon and target depths for the final stage of the attack and throughout each reattack are equal at some depth within the limits of assumption four.

Assumption 7: The attack and each reattack is from astern the target. There is some close-in range at which the weapon sensors lose contact with the target. On losing contact with the target the weapon continues on a straight track at attack speed for a preset dead time. Upon com-

pletion of the dead time run, the weapon instantaneously slows to search speed and commences a left turn at reattack turn rate and constant depth.

Assumption8: The weapon reattack turn is a circle with no advance or transfer. The weapon reacquires the target upon completion of one complete reattack turn and instantaneously increases to attack speed.

Assumption 9: At the time the close-in weapon sensors lose contact with the target, there exists an intersection of the weapon track extension with the vertical plane through the submarine target center which is perpendicular to any plane containing both the target and the weapon. Let the target projection in the vertical plane be approximated by a circle denoted the target circle centered on the target longitudinal axis with a diameter equal to the length of the beam dimension of the target. Let the circle center be the origin of a doubly infinite axis containing the intersection. Assume that the intersection coordinate on the axis is a normally distributed random variable with mean zero and standard deviation σ .

Assumption 10: A hit occurs if the weapon track at the closest point of approach to the target center passes within a distance of the center not greater than the radius of the target circle plus a small proximity correction factor.

Define:

$P_H = P_H(N) = P\left[\text{hit in N attacks}\right]$

$h_n = P\left[\text{hit on the n th attack}\right]$; $n = 1, \cdots, N$

$P_H(0) = 0$

Then

$$P_H = 1 - \prod_{n=1}^{N} (1 - h_n)$$

The derivation is analogous to that for acquisition probability.

Define:

t_o = time the weapon sensors lose contact with the target on an attack or reattack (zero reference time)

RH = range from the weapon to the target center at time t_o (yd)

t_{CPA} = time from t_o of closest point of approach of weapon and target (sec)

MRH = distance traveled by weapon from time t_o to time t_{CPA} (yd)

S_{WA} = weapon attack speed (yd/sec)

S_{TM} = target maximum speed (yd/sec)

BW = target beam width (yd)

VT = weapon proximity correction factor (yd)

r = coordinate of intersection of the weapon track extension with the vertical plane (A) through the submarine target center perpendicular to any plane containing both target and weapon at time t_o (yd)

r' = coordinate of intersection of the weapon with the vertical plane (A') parallel to plane A containing both the submarine target center and weapon at time t_{CPA} (yd)

$F_{\varphi}(x)$ = cumulative normal probability distribution with mean zero and standard deviation one evaluated at the point x

Consider the attack situation depicted in Figure 17.

The xy plane is the vertical containing the weapon and target at time t_o. Planes A and A' are parallel to the yz plane and contain the

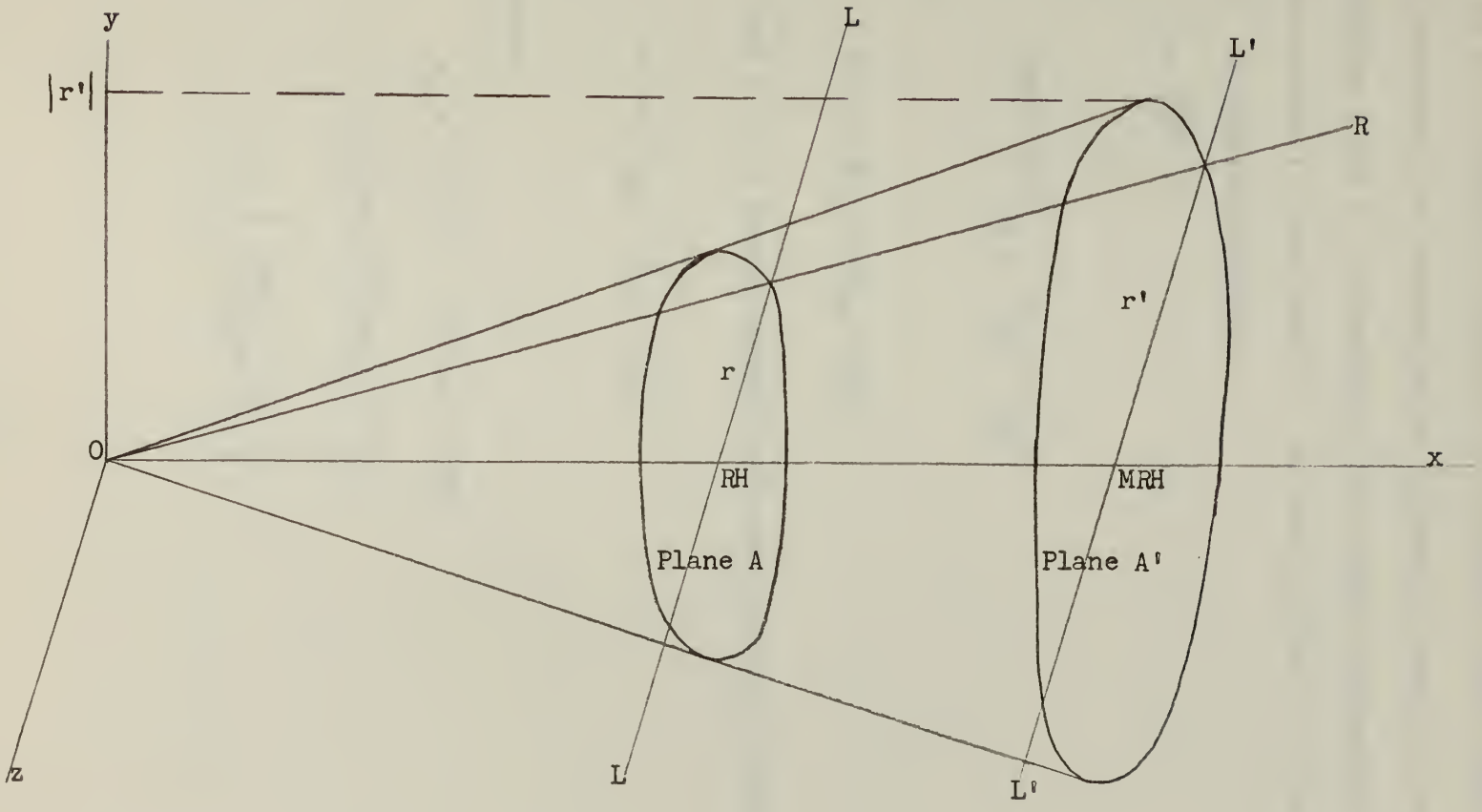


Figure 17: Planes A and A' in the attack situation.

bases of two right circular cones with a common apex at the origin. The line OR is the weapon track and its orientation with respect to the x axis determines the orientation of the axes LL and $L'L'$ on which coordinates r and r' are measured. The common intersection of the cone of base radius r', the plane A', and the xy plane is a line of length $2|r'|$. By similar triangles

$$r' = \frac{MRH}{RH} \; r$$

Since

$$MRH = RH \; S_{WA} \; / \; (S_{WA} - S_{TM})$$

then

$$r' = r \; S_{WA} \; / \; (S_{WA} - S_{TM})$$

Thus r' is a random variable with mean zero and standard deviation σ'.

$$\sigma' = \sigma \; S_{WA} \; / \; (S_{WA} - S_{TM})$$

Therefore

$$h_n = P\left[2\,|r'| \leq BW + 2\,VT\right]$$

$$= P\left[|r'| \leq (BW/2) + VT\right]$$

$$= 2\,F_{\phi}\left[\frac{(BW/2) + VT}{\sigma'}\right] - 1$$

$$h_n = 2 F_\phi \left[\frac{(BW/2) + VT}{\sigma S_{WA} / (S_{WA} - S_{TM})} \right] - 1$$

Define:

HD = hit depth (ft)

$f_{HD}(x)$ = probability density function of the random variable hit depth

D_{TM} = maximum target depth (ft)

D_{MIN} = minimum target depth to be considered (ft)

Define a set of Q equal depth subintervals in the depth interval $\left[D_{MIN}, D_{TM}\right]$ such that the depth at the lower boundary of the q th subinterval is $D_{MIN} + q \left[\frac{D_{TM} - D_{MIN}}{Q}\right]$. Define

$P_{Eq} = P\left[\text{hit depth is in the q th depth subinterval}\right]$; q=1, ···, Q

Assume as in Figure 18:

$$f_{HD}(x) = 2 (x - D_{MIN}) / (D_{TM} - D_{MIN})^2$$

Then:

$$P_{Eq} = \frac{2q - 1}{Q^2}; \quad q = 1, \cdots, Q$$

Define:

w'_{2n} = fuel used in the n th attack or reattack (lb)

R_A = initial acquisition range (yd)

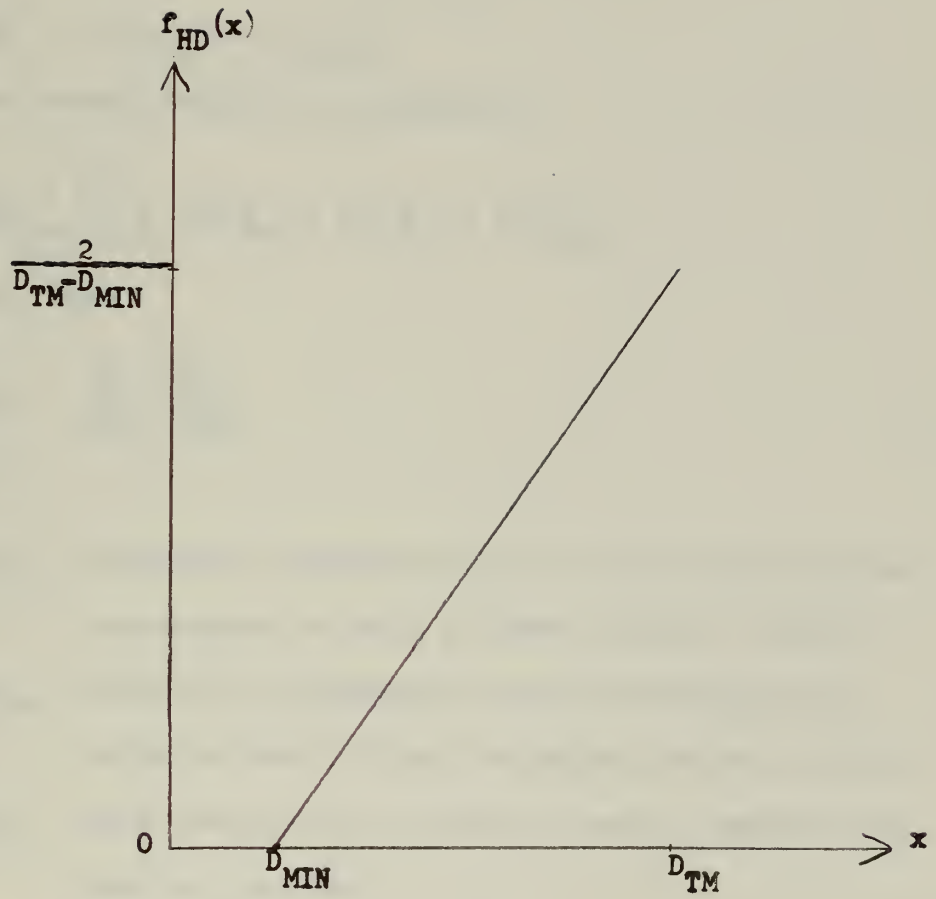


Figure 18: Assumed probability density function of the random variable hit depth.

w_{2t} = weapon fuel expenditure rate at attack speed and at the average depth for the attack. (lb/sec)

$\overline{HD}$ = average hit depth (ft)

Then

$$\overline{HD} = 2/3\ D_{TM} + 1/3\ D_{MIN}$$

and the average depth for the attack is

$$\frac{D_{WS} + \overline{HD}}{2} = 1/2\ D_{WS} + 1/3\ D_{TM} + 1/6\ D_{MIN}$$

For the attack

$$W'_{21} = \frac{R_A \cdot w_{2t}}{S_{WA} - S_{TM}}$$

Define:

w_{Stq} = weapon fuel expenditure rate at search speed and at the midpoint of the q th depth interval. (lb/sec)

w_{Atq} = weapon fuel expenditure rate at attack speed and at the midpoint of the q th depth interval. (lb/sec)

t_{DT} = dead time from t_o at which the weapon commences the reattack turn (sec)

t_{CT} = time from t_o at which the weapon completes the reattack turn and reacquires the target (sec)

ω_R = reattack rate of turn (deg/sec)

Then:

$$t_{CT} = t_{DT} + \frac{360^o}{\omega_R}$$

$$t_{CPA} = \frac{RH}{S_{WA} - S_{TM}}$$

The weapon will spend the following time at attack speed during each reattack: See Figure 19.

$$T_A = t_{DT} - t_{CPA} + \frac{(T_{CT} - t_{CPA})\, S_{TM} - (t_{DT} - t_{CPA})\, S_{WA}}{S_{WA} - S_{TM}}$$

The weapon will spend the following time at search speed during each reattack:

$$T_S = \frac{360^o}{\omega_R}$$

Therefore:

$$w'_{2n} = \sum_{q=1}^{Q} \left[T_A\, w_{Atq} + T_S\, w_{Stq} \right] P_{Eq}$$

$$n = 2, \cdots, N$$

Define:

S_o = minimum preset value of attack speed (yd/sec)

w'_2 = available fuel weight for hit, event 2, (lb)

w''_2 = available weight for modifications to the propulsion system to provide a maximum speed capability in excess of S_o (lb)

N is a function of w'_2 according to

$$\sum_{n=0}^{N} w'_{2n} \leq w'_2 < \sum_{n=0}^{N+1} w'_{2n}$$

However, recall that w'_{2n} is a function of attack speed which is itself a function of w''_2.

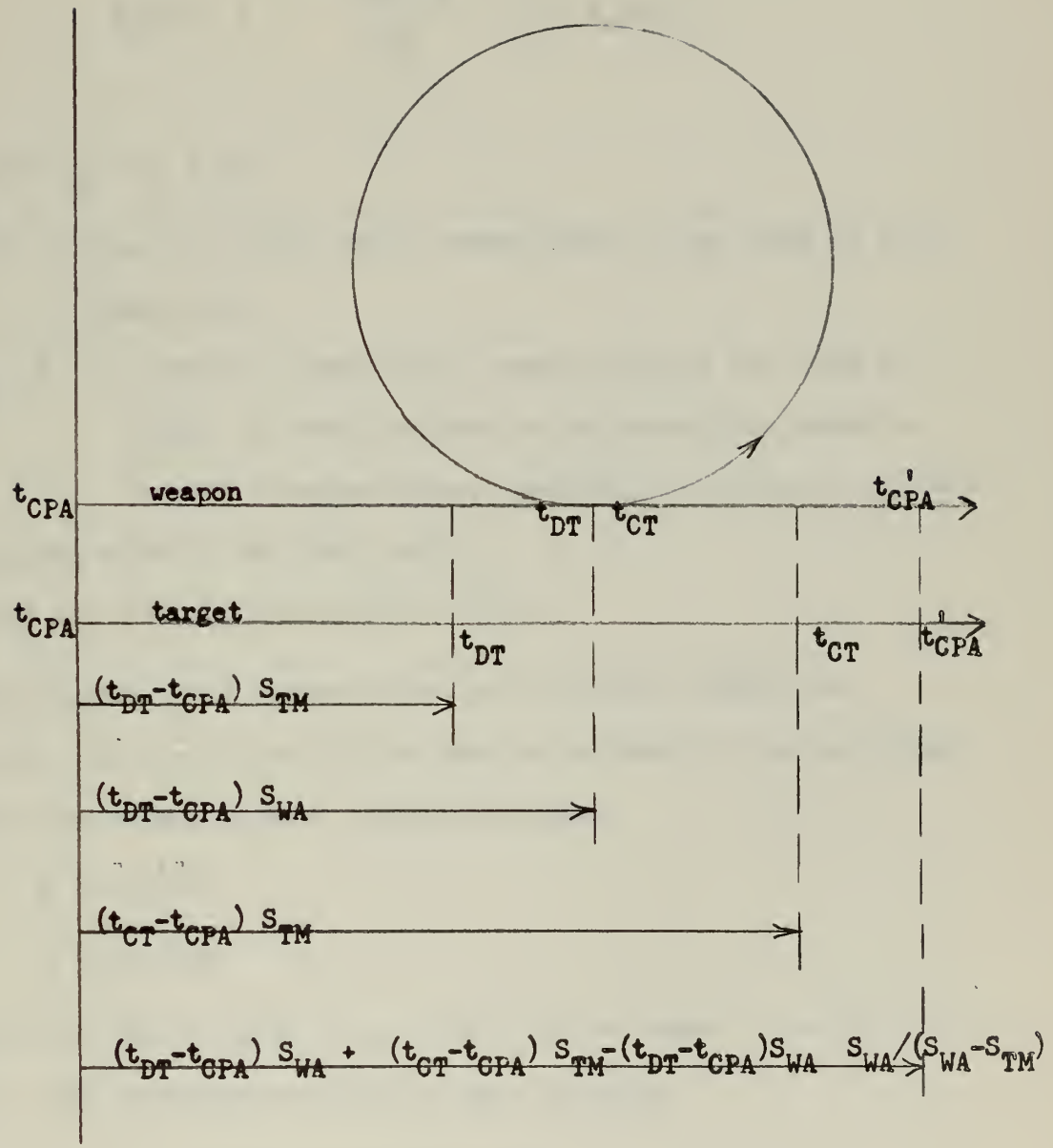


Figure 19: Geographic plot of weapon and target tracks separated to show time differences. The distancesbelow the target track are measured from the joint position at time t_{CPA}.

Thus

$$P_H(N) = 1 - \prod_{n=1}^{N(w_2', w_2'')} \left[1 - h_n(w_2'')\right]$$

Recall $w_2 = w_2' + w_2''$

$\underline{W}', \bar{W}'$ = lower and upper limits respectively on the range of fuel weight (lb)

$\underline{W}'', \bar{W}''$ = lower and upper limits respectively on the range of weight for modifications to the propulsion system to provide a maximum speed capability in excess of S_o (lb)

For each value in the range for w_2

$$\min\left\{\underline{W}', \underline{W}''\right\} \leq w_2 \leq \min\left\{\bar{W}' + \bar{W}'', W\right\}$$

the function P_H is evaluated for each possible combination of the variables w_2' and w_2'' as they are allowed to vary such that they remain within their respective ranges

$$\underline{W}' \leq w_2' \leq \bar{W}'$$

$$\underline{W}'' \leq w_2'' \leq \bar{W}''$$

and their sum is equal to w_2. Let K be the number of values of w_2. This procedure provides J_k sets of values

$$w_{2jk}', w_{2jk}'', P_{Hjk}, \quad j = 1, \cdots, J_k$$

for the k th value of w_{2k}. $(k = 1, \cdots, K)$

The entire set of sets of values

$$w_{2k}, w_{2jk}', w_{2jk}'', P_{Hjk}; \quad j = 1, \cdots, J_k$$

$$k = 1, \cdots, K$$

is the input to the dynamic programming allocation model.

Input Requirements

1. Target

BW = target beam width (yd)

D_{TM} = target maximum depth (ft)

S_{TM} = target maximum speed (yd/sec)

2. Weapon

S_{WS} = weapon search speed (yd/sec)

VT = weapon proximity correction factor (yd)

ω_R = weapon reattack turn rate (deg/sec)

$\underline{W}'$, $\bar{W}'$ = lower and upper limits respectively on the allowable range of fuel weight (lb)

S_o = minimum allowable weapon attack speed (yd/sec)

$\underline{W}''$, $\bar{W}''$ = lower and upper limits respectively on the range of weight for modifications to the propulsion system to provide a maximum speed capability in excess of S_o (lb)

G_1 = Relationship between increased weapon speed above S_o and increased weapon weight

G_2 = Relationship between fuel expenditure rate and speed and depth. Recall that this relationship may vary as a function of attack speed only in the range for speeds above search speed. This relationship is required to specify w_{Stq}, w_{Atq}, and w_{2t} (lb/sec)

3. Miscellaneous

RH = range from the weapon to the target center at the instant (t_o) during close-in attack when the weapon sensors lose contact with the target (yd)

σ = standard deviation of the normally distributed random variable: coordinate of intersection of the weapon track extension with the vertical plane (A) through the submarine target center perpendicular to any plane containing both target and weapon at time t_o . (yd)

D_{MIN} = minimum target depth to be considered (ft)

R_A = initial acquisition range (yd)

Q = number of depth subintervals in the depth interval $\left[D_{MIN}, D_{TM}\right]$

Intelligence reports and estimates or CNO requirements may provide the target inputs. The weapon inputs are available from weapon designers at least in the form of educated estimates. The inputs RG and σ are functions of the medium, the weapon sonar and guidance systems, the target physical dimensions, and the orientation relationships between the target and weapon. In the absence of a specific theory these must be handled by simulation or sensitivity analysis. The input D_{MIN} is a function of the capability of the weapon delivery system and the tactical situation as seen by the target commander. In the absence of specific considerations criterion one would determine D_{MIN} to be zero. The input R_A should be

selected close to the maximum weapon capability in accordance with criterion one. The function P_H may be very sensitive to changes in R_A and an examination of results over a range of values for R_A may be appropriate. The input Q should be large enough to point up the effect of an irregularly shaped fuel expenditure rate curve but small enough not to excessively prolong the computer program.

CHAPTER V

PROBABILITY OF KILL DAMAGE

Consider a tactical situation in which a weapon explodes within a specified proximity of some portion of a submarine target and make the following assumptions.

Assumption 1: The hit depth is a random variable which has an assumed probability density function.

Assumption 2: The point on the target at which the hit occurs denoted the hit point is a random variable which has a uniform distribution over the entire surface area of the target. The weapon approaches the target from astern, loses contact at some close-in range and continues on a straight track. Since the target may be in a turn, climb or dive, all points of the target are available as hit points.

Assumption 3: The target operates in depth within specified minimum and maximum limits such that the maximum depth limit is not greater than the weapon maximum operating depth. Divide the target surface area into P area elements, each of which has a constant hull vulnerability throughout. Divide the interval of possible hit depths into Q equal depth subintervals such that the depth at the lower boundary of the q th subinterval is

$$D_{MIN} + q \left[\frac{D_{TM} - D_{MIN}}{Q} \right]$$

where

D_{TM} = maximum target depth (ft)

D_{MIN} = minimum target depth to be considered (ft)

Define: $P_E = P_E(w_3) = P$ [kill damage given an explosive weight w_3]

$f_{HD}(x)$ = probability density function of the random variable hit depth

$P_{Eq} = P$ [hit depth is in the q th depth subinterval]; $q = 1, \cdots, Q$

$P_{Ep} = P$ [hit point is in the p th surface area element]; $p = 1, \cdots, P$

$HV_{pq}(w_3)$ = hull vulnerability for kill damage of the p th target surface area element at the midpoint depth of the q th depth subinterval given an explosive weight w_3. This number is one if kill damage occurs under the specified conditions and is zero if kill damage does not occur.

w_3 = explosive weight (lb)

$\underline{W}'''$, $\bar{W}'''$ = lower and upper limits respectively on the range of explosive weight (lb)

Assuming as in Chapter IV

$$f_{HD}(x) = \frac{2(x - D_{MIN})}{(D_{TM} - D_{MIN})^2}$$

Then

$$P_{Eq} = \frac{2q - 1}{Q^2}; \quad q = 1, \cdots, Q$$

$$P_{Ep} = \frac{\text{area of p th surface area element}}{\text{total target surface area}}; \quad p = 1, \cdots, P$$

$$P_E = P_E(w_3) = \sum_{p=1}^{P} \sum_{q=1}^{Q} HV_{pq}(w_3) P_{Ep} P_{Eq}$$

The table of P_E versus w_3 as w_3 varies from $\underline{W}'''$ to $\bar{W}'''$ is the input to the dynamic programming model.

Input Requirements

1. Target

   $P_{Ep} = P$ [hit point is in the p th surface area element]; $p = 1, \cdots, P$

   D_{TM} = maximum target depth (ft)

2. Weapon

   $\underline{W}'''$, $\bar{W}'''$ = lower and upper limits respectively on the range of explosive weight (lb)

3. Target and Weapon

   $HV_{pq}(w_3)$ = hull vulnerability for kill damage of the p th target area element at the midpoint of the q th depth subinterval given an explosive weight (w_3)

4. Miscellaneous

   D_{MIN} = minimum target depth to be considered (ft)

   Intelligence or CNO requirements may provide D_{TM}, P_{Ep} and some of the information for $HV_{pq}(w_3)$. Weapon designers may estimate $\underline{W}'''$ and $\bar{W}'''$ and weapons engineers may provide extrapolated estimates of $HV_{pq}(w_3)$.

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2. The Influence of Component Characteristics and Target Parameters on the Size and Weight of Torpedoes by R. C. Brumfield, NOTS, China Lake, Calif., 7 May 1956. (NOTS 1450, NAVORD REPORT 5248) CONFIDENTIAL

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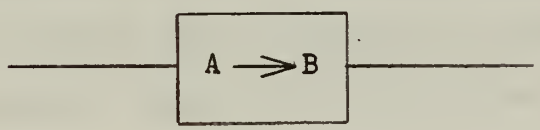
APPENDIX A

FLOW CHART NOTATION

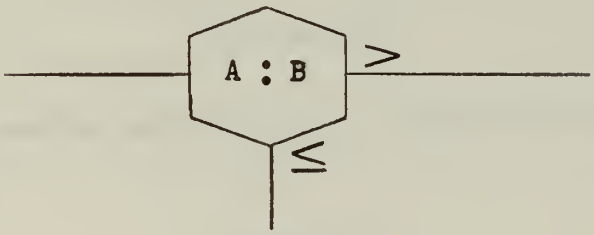
The following notation is used in the flow charts of the remaining appendices.

Function Box



The function box contains a number or expression to be evaluated A which is to be placed in memory cell B.

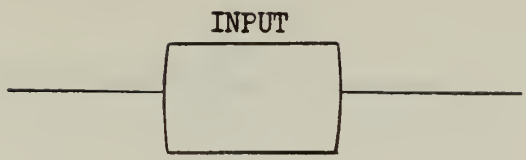
Choice Box



The choice box contains a comparison of the number or expression A to the number or expression B. Exits are chosen as follows:

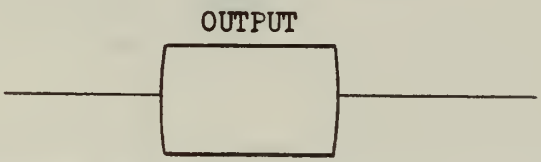
Exit Label	Exit is chosen if
$>$	$A > B$
$\geq$	$A \geq B$
$=$	$A = B$
$<$	$A < B$
$\leq$	$A \leq B$

In Box



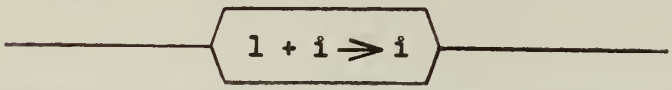
The in box contains an input to the program. In appendices C 1 through C 8 the output from one appendix is carried over intact into the next appendix. Inputs to be used for the first time in the sequence of flow charts are placed in a separate in box.

Out Box



The out box, one for each appendix, contains the output of the appendix program.

Index Box



The index box contains a manipulation on an integer in an index register and is the tool which provides for loops in the program.

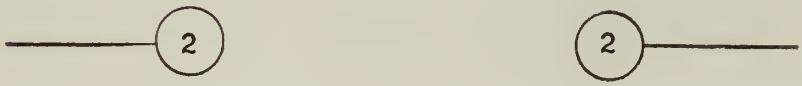
Subroutine Box

The subroutine box indicates a return jump to the subroutine as numbered by k. The flow chart of each subroutine is shown after the flow chart of the first routine using it.

Return Box

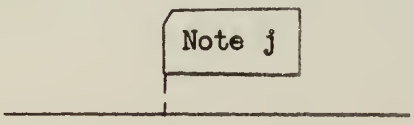
A return box indicates the end of a subroutine implying a jump back to the main routine.

Fixed Connector



A fixed connector is used to connect remote parts of the flow chart with one another.

Assertion Box

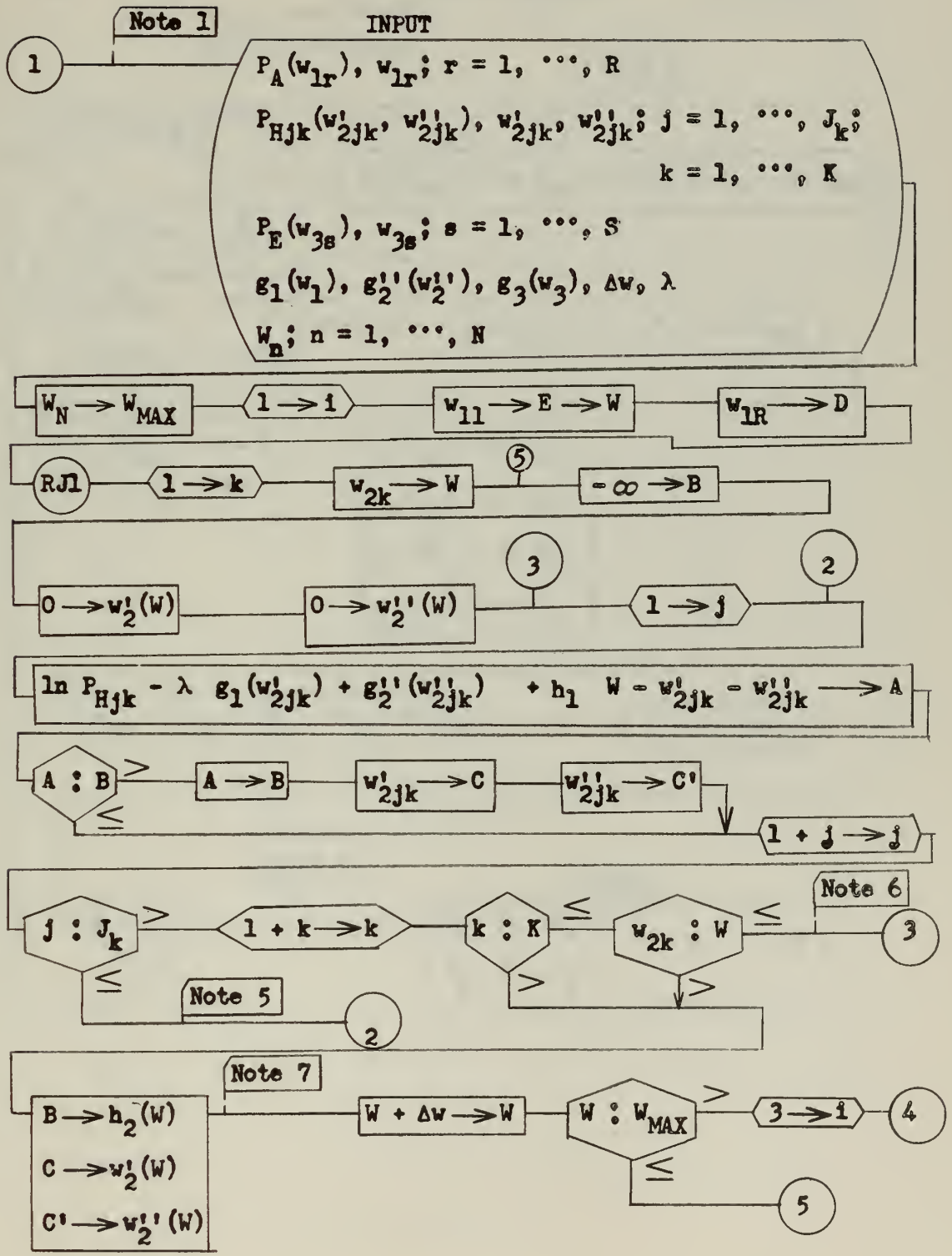


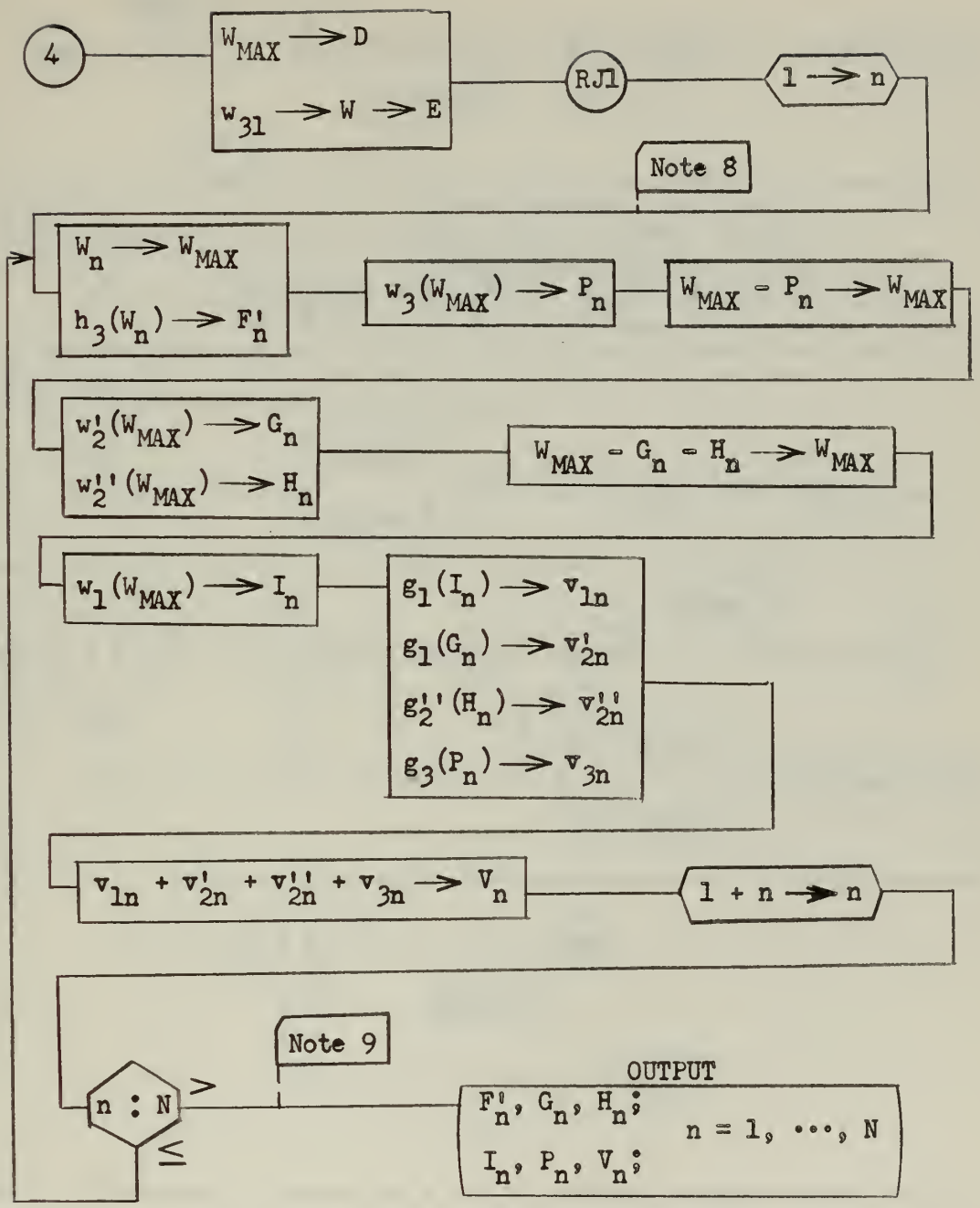
An assertion box refers the reader to a note found at the end of the flow chart.

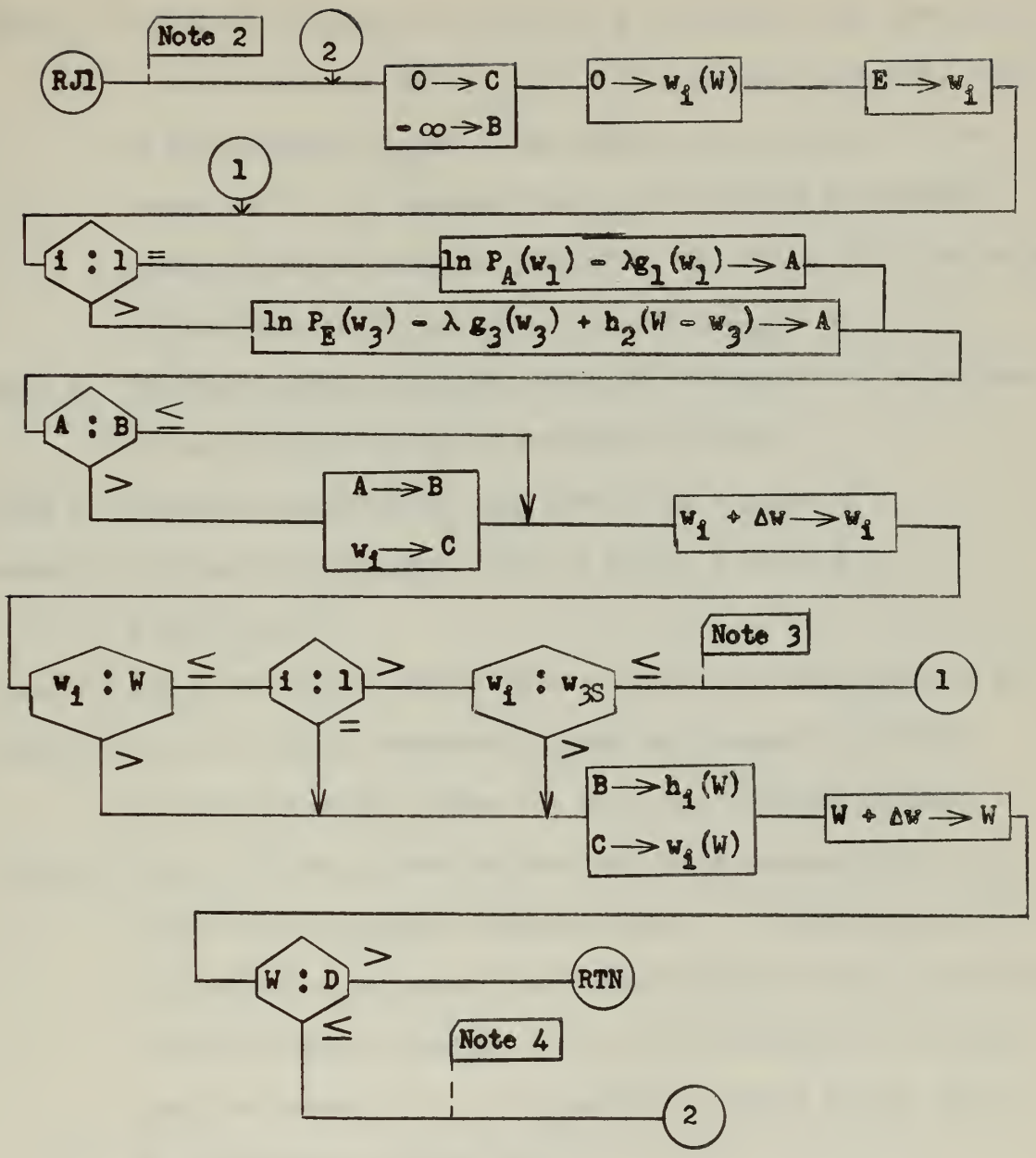
Instruction flow is generally from left to right and from top to bottom. All boxes are entered from the left or from the top and all exits from boxes are made down or to the right. The arrow is used to further indicate the sequence of computer instructions. Arrows are used at all line intersections to indicate entry into the program between two boxes.

APPENDIX B

THE ALLOCATION MODEL




4
$W_{MAX} \rightarrow D$
$w_{31} \rightarrow W \rightarrow E$
RJ1
$1 \rightarrow n$
Note 8
$W_n \rightarrow W_{MAX}$
$h_3(W_n) \rightarrow F'_n$
$w_3(W_{MAX}) \rightarrow P_n$
$W_{MAX} - P_n \rightarrow W_{MAX}$
$w'_2(W_{MAX}) \rightarrow G_n$
$w''_2(W_{MAX}) \rightarrow H_n$
$W_{MAX} - G_n - H_n \rightarrow W_{MAX}$
$w_1(W_{MAX}) \rightarrow I_n$
$g_1(I_n) \rightarrow v_{1n}$
$g_1(G_n) \rightarrow v'_{2n}$
$g''_2(H_n) \rightarrow v''_{2n}$
$g_3(P_n) \rightarrow v_{3n}$
$v_{1n} + v'_{2n} + v''_{2n} + v_{3n} \rightarrow V_n$
$1 + n \rightarrow n$
Note 9
n : N
>
≤
OUTPUT
$F'_n, G_n, H_n;$
$I_n, P_n, V_n;$
$n = 1, \cdots, N$



Note 1: Provision is made for a set of payload weight limits (W_n) in case it is desired to examine a range of values. The input W_N is to be the greatest of the W_n.

Note 2: The subroutine RJ1 computes the $h_i(W)$ and the $w_i(W)$ for i=1, 3. h_2 is handled separately.

Note 3: This loop provides that, for the W considered, the function h_i is maximized over the w_i. For h_1 the maximum meaningful value in both loops is $w_{1R}=D$ so the subroutine is stopped by the comparison W : D. However for h_3, the maximum meaningful value in the two loops is different, the one in this loop being w_{3S} and the one in the other loop being $W_{MAX} = D$.

Note 4. This loop stores the $h_i(W)$ and $w_i(W)$ obtained as W is allowed to vary over its range of meaningful values.

Note 5: This loop maximizes the w_{2jk} over j for a given k.

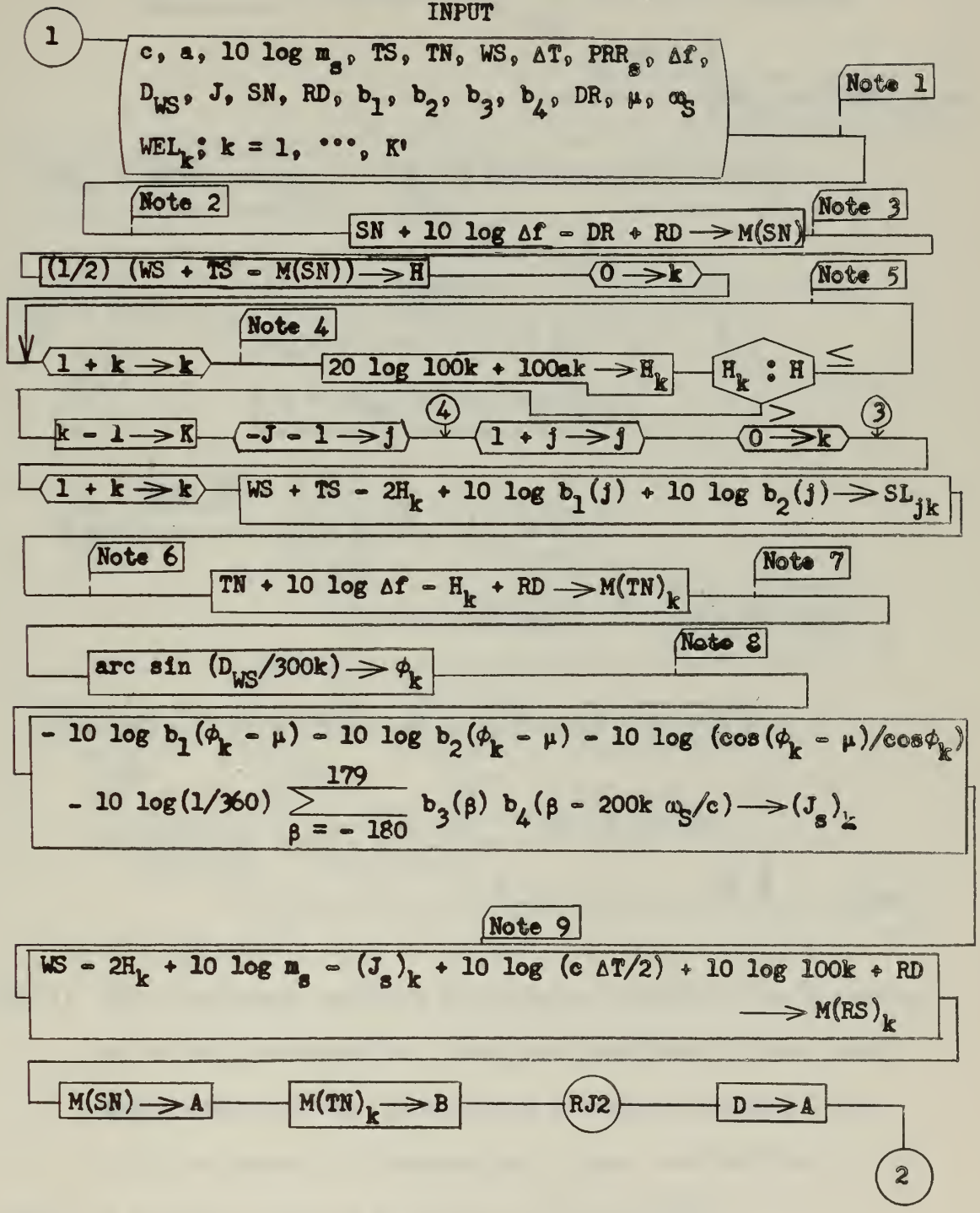
Note 6: The maximum meaningful value of w_2 for a given W is $\min \{w_{2K}, W\}$.

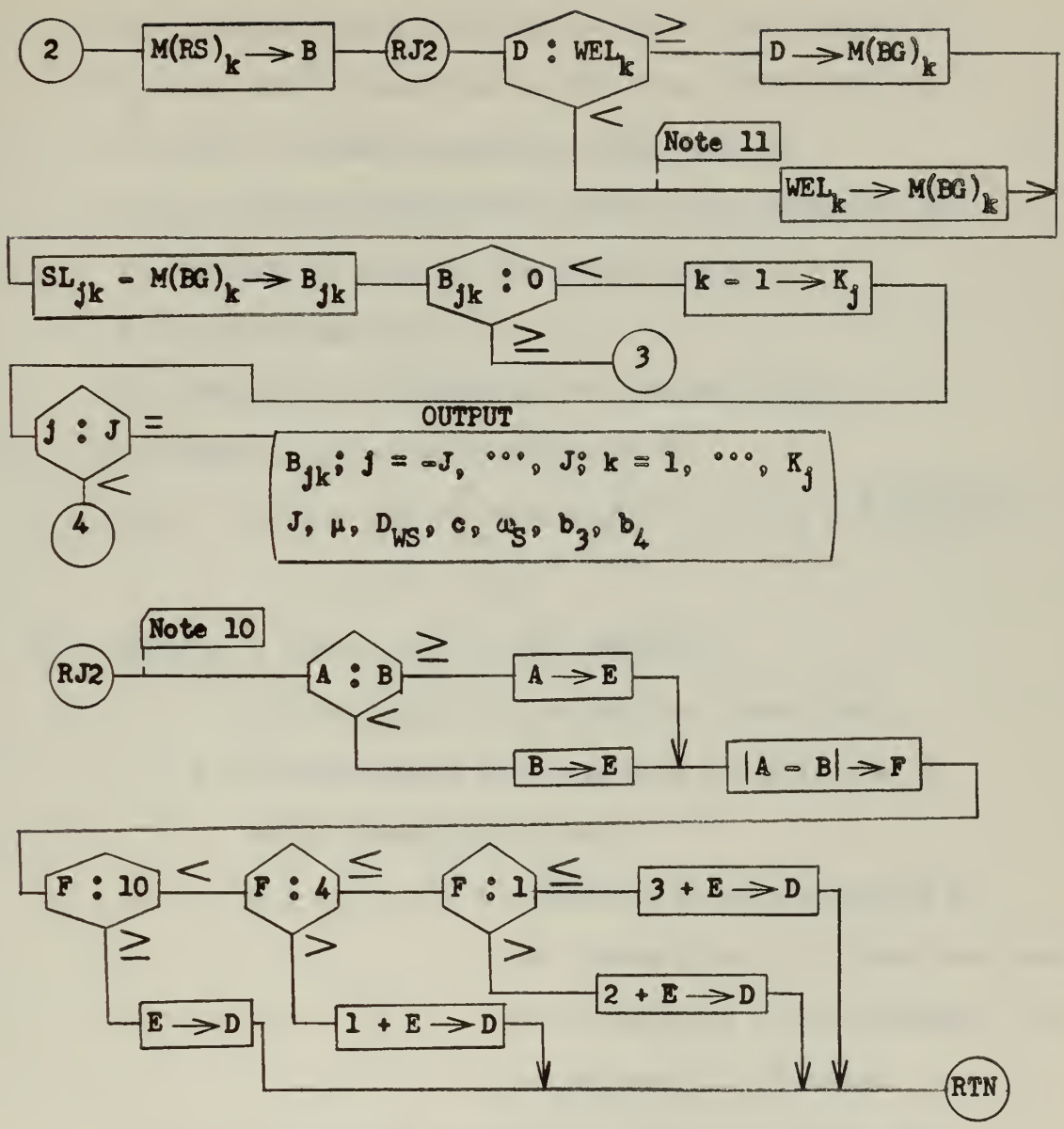
Note 7: The $h_2(W)$, $w_2'(W)$, and $w_2''(W)$ are stored for each value of W.

Note 8: This loop works backwards through the stages to retrieve the optimum weight values for each input payload maximum.

Note 9: The V_n are the optimum volume sums to be compared with the input payload maximum allowed volume V. If the solution is not optimal with respect to the restraints, a new λ is chosen and the program repeated. If the final solution is obtained, then fuel weight is $G_n + I_n$, weight for added attack speed is H_n and explosive weight is P_n.

APPENDIX C1
AVAILABLE ECHO EXCESS (β_{jk})





Note 1: This flow chart neglects the effects of ambient sea noise and of volume reverberation. Both are considered to have negligible effect on the interference background masking level.

If it is desired to consider the volume reverberation, it is

$$M(R_v)_k = WS - 2H_k + 10 \log m_v - (J_v)_k + 10 \log \left(\frac{c \Delta T}{2}\right)$$

$$+ 20 \log (100k) + RD; \; k \; 1, \cdots, K$$

where $M(R_v)_k$ = volume reverberation masking level at range 100k yards (db)

WS = on-axis source level (db at 1 yard from transducer)

H_k = one way transmission loss at range 100k yards (db)

10 log m_v = volume scattering coefficient (db)

$(J_v)_k$ = volume reverberation index of the transducer (db)

c = velocity of sound in the medium (yd/sec)

ΔT = ping length (sec)

RD = recognition differential of transducer (db)

The index $(J_v)_k$ may be approximated by

$$(J_v)_k = -10 \log \frac{1}{4\pi} \int_{\Omega} b_1(j)\, b_2(j)\, b_3(\beta)\, b_4\left(\frac{\beta - 200\, k\omega_s}{c}\right) d\Omega$$

$$k = 1, \cdots, K$$

where ω_s = search rate of turn (deg/sec)

j = off-axis angle in the vertical plane (deg)

β = off-axis angle in the azimuth plane at time of pulse transmission (deg)

b_1, b_2, b_3, b_4 = normalized pattern functions of the transmitting (1, 3) and receiving (2, 4) patterns in the vertical (1,2) and azimuth (3, 4) planes

Ω = solid angle subtended by the sound energy beam. This in effect assumes a medium of infinite extent in the direction of the sound beam.

Note 2: $M(SN) = SN + 10 \log \Delta f - DR + RD$

where $M(SN)$ = self-noise masking level (db)

SN = equivalent isotropic self-noise level at search speed and depth (db)

Δf = receiving bandwidth (cycles/sec)

DR = receiving directivity index (db)

Note 3: $2H = WS + TS - M(SN)$

where H = maximum possible on-axis range for acquisition (yd). This neglects noise levels except the self-noise level and neglects the weapon enabling level.

TS = target strength (db)

Note 4: $H_k = 20 \log 100k + 100 ak$

where a = attenuation coefficient (db/yard) and it is assumed that the transmission anomaly is zero.

Note 5: This loop finds the upper limit (100K) on the acquisition ranges to be considered.

Note 6: $M(TN)_k = TN + 10 \log \Delta f - H_k + RD;\ k=1, \ldots, K$

where $M(TN)_k$ = target noise masking level (db)

TN = radiated noise level of the target (db at 1 yard from transducer)

Note 7: $\phi_k = \arcsin \frac{D_{WS}}{300k}$

where ϕ_k = the angle above the horizontal at which an arc of radius 100k yards and of origin at search depth will intersect the surface of the water

D_{WS} = weapon search depth (ft)

Note 8: $(J_s)_k$ = surface reverberation index of the transducer (db)

μ = angle of attack (pitch) of the weapon (deg)

This is an approximation described in Chapter 4 of reference 4.

Note 9: $M(RS)_k$ = surface reverberation masking level at a range of 100k yards (db)

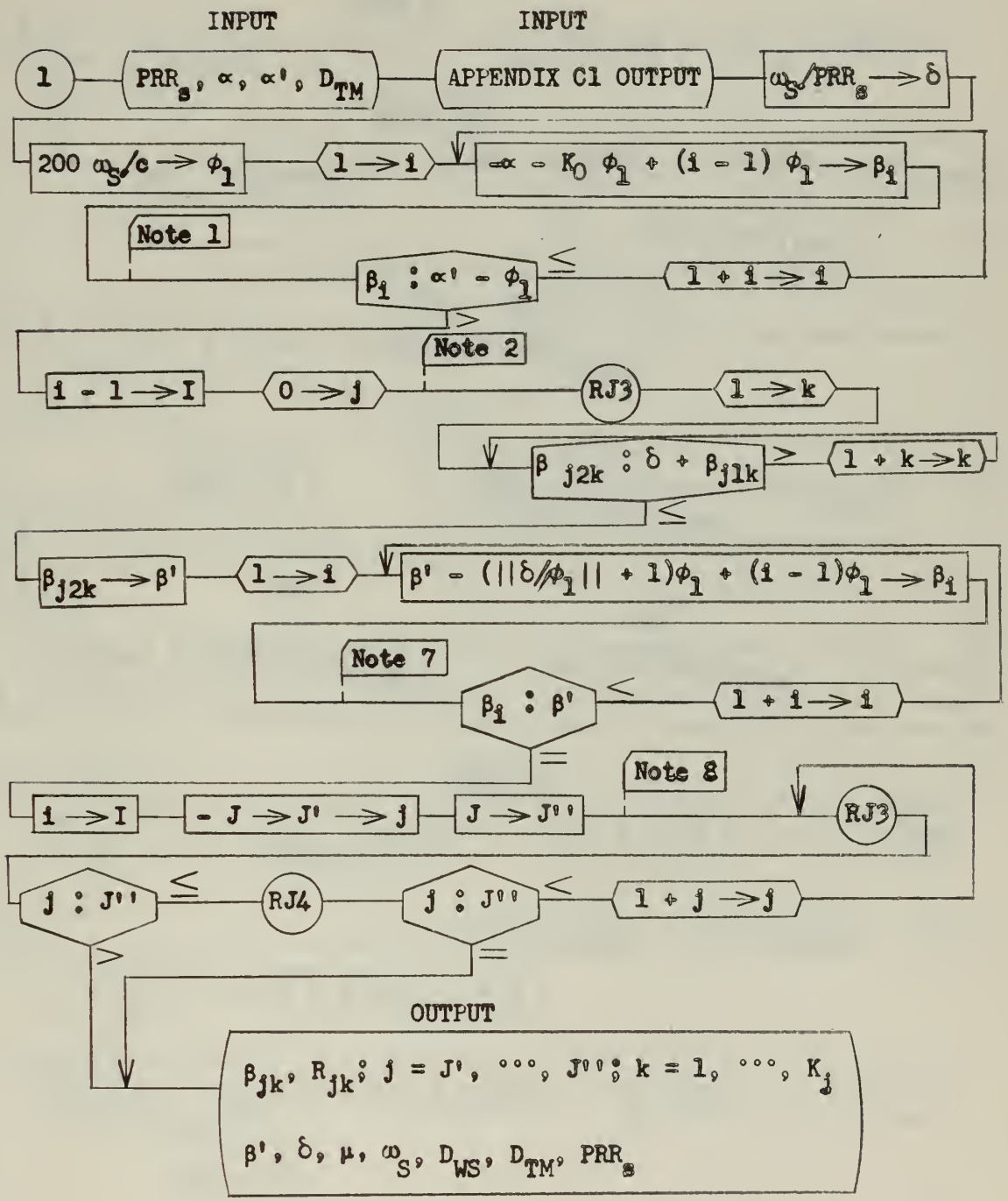
$10 \log m_S$ = surface reverberation scattering coefficient (db)

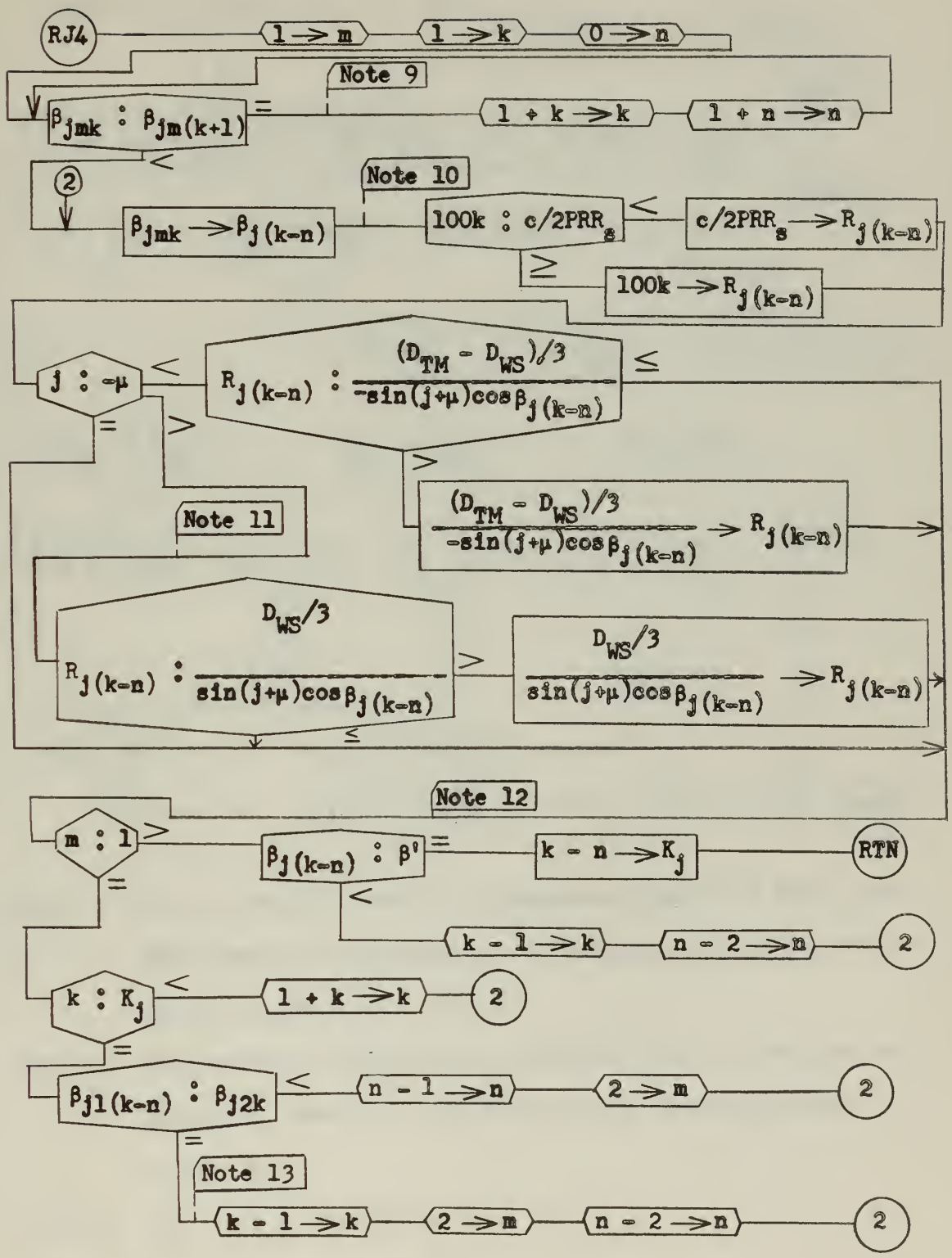
Note 10: This subroutine adds the effects of two noise levels in decibels. A rounding off process is employed based on Figure 3 of reference 3. A difference in levels of more than 10 db provides a sum effect equal to the larger of the individual levels. Two equal levels have a sum effect equal to 3 db higher than either individual level.

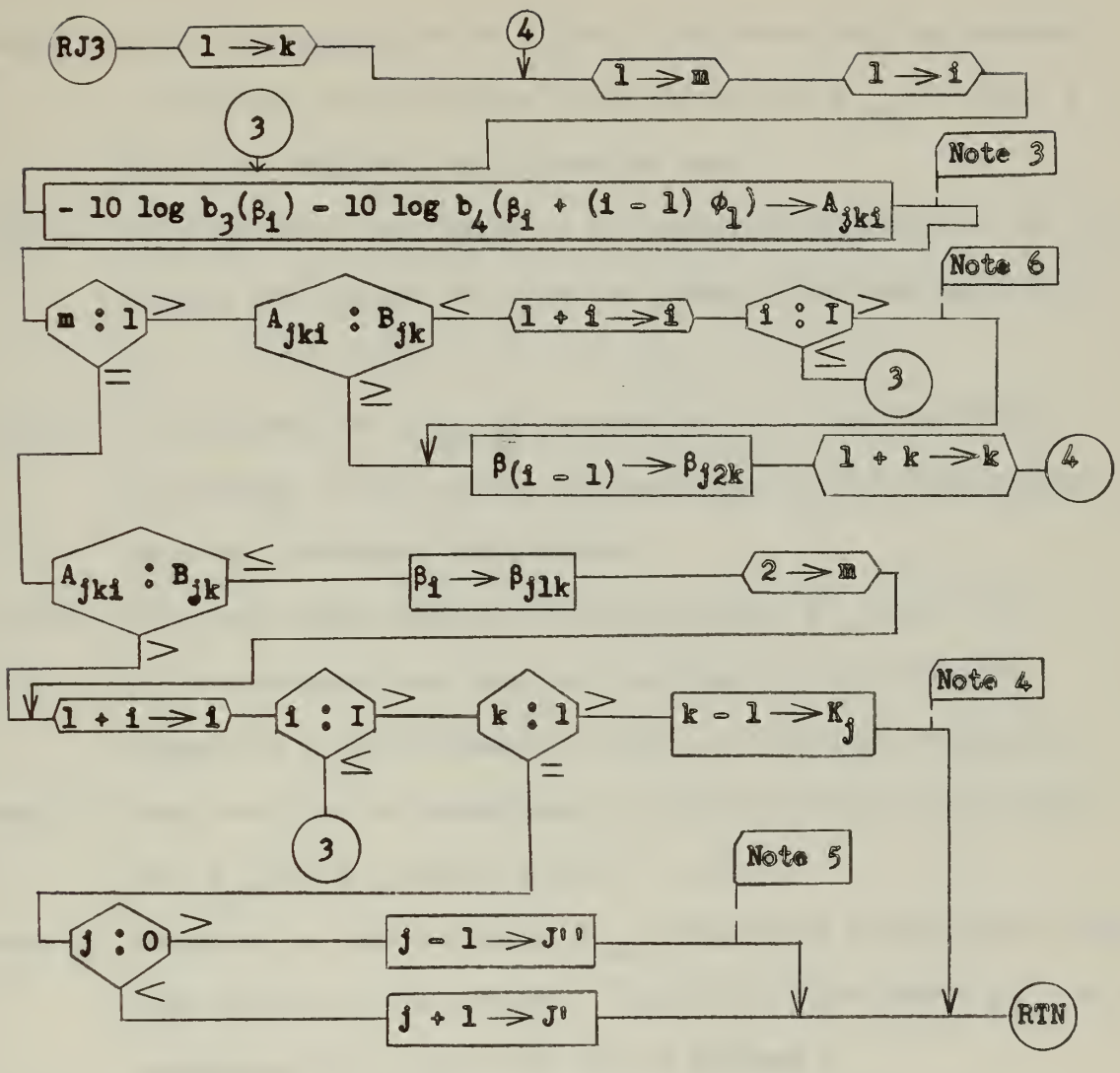
Note 11: WEL_k = weapon enabling level (db). Values out to a range of 100K' yards are inputs where K' is estimated such that it will be larger than the K found in the program.

APPENDIX C2

SCANNED VOLUME RADIALS; GENERAL CASE




RJ4
1 → m
1 → k
0 → n
Note 9
$\beta_{jmk} : \beta_{jm(k+1)}$
=
1 + k → k
1 + n → n
<
2
Note 10
$\beta_{jmk} \rightarrow \beta_{j(k-n)}$
100k : c/2PRR$_s$
<
c/2PRR$_s$ → R$_{j(k-n)}$
≥
100k → R$_{j(k-n)}$
j : -μ
<
R$_{j(k-n)}$: $\frac{(D_{TM} - D_{WS})/3}{-\sin(j+\mu)\cos\beta_{j(k-n)}}$
≤
=
>
Note 11
$\frac{(D_{TM} - D_{WS})/3}{-\sin(j+\mu)\cos\beta_{j(k-n)}} \rightarrow R_{j(k-n)}$
R$_{j(k-n)}$: $\frac{D_{WS}/3}{\sin(j+\mu)\cos\beta_{j(k-n)}}$
>
$\frac{D_{WS}/3}{\sin(j+\mu)\cos\beta_{j(k-n)}} \rightarrow R_{j(k-n)}$
≤
Note 12
m : 1
>
$\beta_{j(k-n)} : \beta'$
=
k - n → K$_j$
RTN
=
<
k - 1 → k
n - 2 → n
2
k : K$_j$
<
1 + k → k
2
=
$\beta_{j1(k-n)} : \beta_{j2k}$
<
n - 1 → n
2 → m
2
=
Note 13
k - 1 → k
2 → m
n - 2 → n
2



Note 1: The β_i are all possible transmission angles in plane zero which meet the requirement that the echo will be received in the interval $\left[-\alpha, \alpha'\right]$.

Note 2: The overlap in plane zero is computed first to provide the angle β'. Then $\beta' - \delta$ and β' define the limiting angles of the β_{jmk}.

Note 3: $A_{jki} = A_{jk}(\beta)$ for the i th β.

Note 4: After the computation of K_j which indicates that the maximum acquisition range in plane j and all of the β_{jmk} in plane j have been computed, the subroutine ends.

Note 5: This indicates that there is no acquisition probability in the plane j and changes the range of planes to be considered from -J, ···, J to J', ···, J''.

Note 6: In this case the β_{j2k} is limited by the β' and would have been larger if the overlap between pings had not been removed from the considered scanned area.

Note 7: This loop starts with the leftmost angle ($\beta' - (I-1)\ \phi_1$) to be considered and computes each angle to be considered indexed by i and incremented by ϕ_1, to a maximum angle $\beta_I = \beta'$.

Note 8: This point is the commencement of the loop which will compute the β_{jk} and R_{jk} starting with j = -J.

Note 9: There may be several equal β_{jmk} because the scan azimuth limits were reduced by the overlap. The first of the equal β_{jmk} to be considered will then be the one of maximum k.

Note 10: The ping interval range is $\frac{C}{2PRR_s}$ yards/ping.

Note 11: The range at which the radial at angle β_{jmk} intersects the surface of the water for $j > -\mu$ is computed below.

See Figure 20. The distance OC is desired. The search depth is (DWS/3) yards.

length (CB) = $D_{WS} / 3 \sin(j+\mu)$

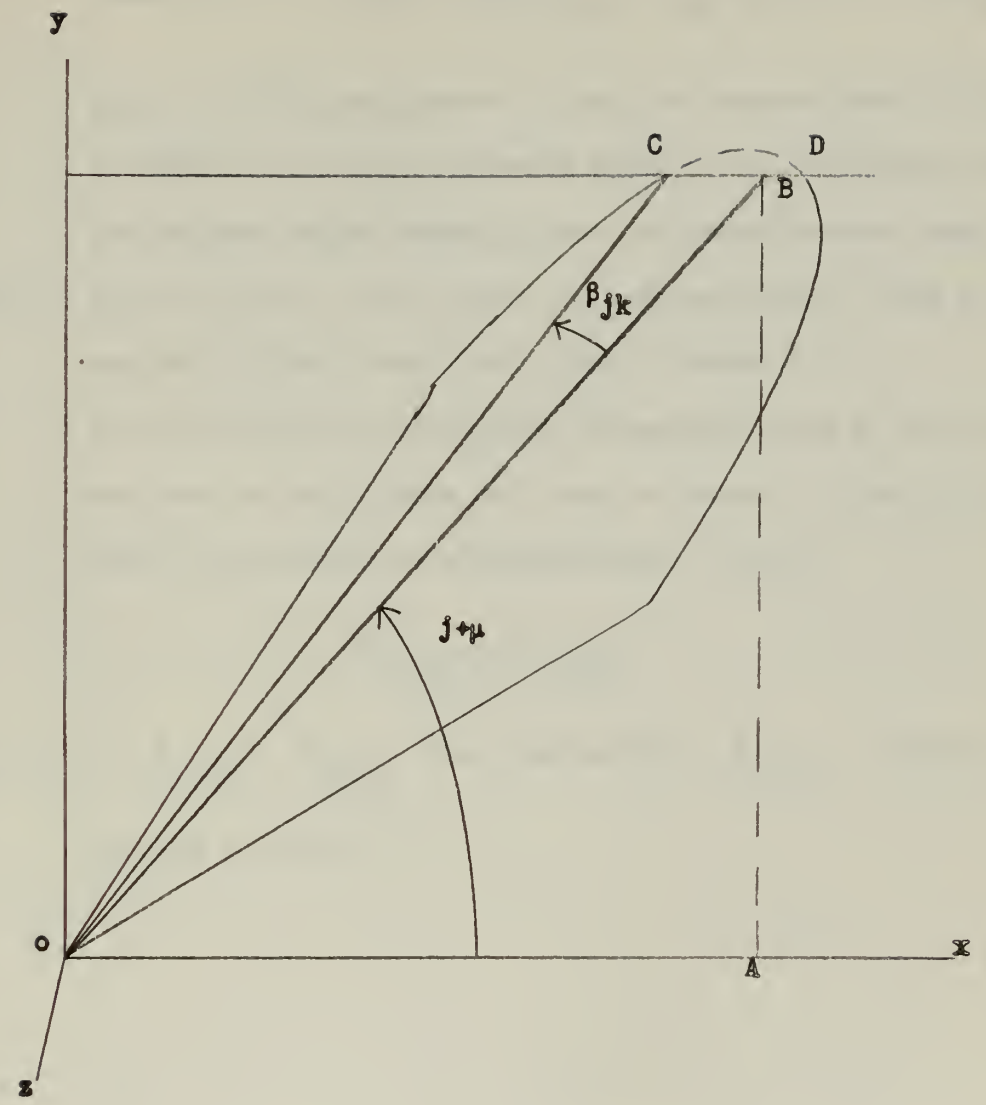


Figure 20. Computation of radial length for angle β_{jk}. The plane j for which the scan outline is shown is perpendicular to the vertical xy plane. Angles CBO and DBO are right angles. Length AB is the search depth of $D_{WS}/3$ yards.

$$\text{length (OC)} = \text{length (OB)}/\cos\beta_{jk} = D_{WS}/3 \sin(j+\mu) \cos\beta_{jk}$$

For $j = -\mu$ the development is the same except that $(j + \mu)$ is negative and the considered depth is the difference between the maximum target depth D_{TM} and the weapon search depth.

Note 12: If to the left of the radial of maximum range K_j then $m = 1$ and the following angle will have a larger k.

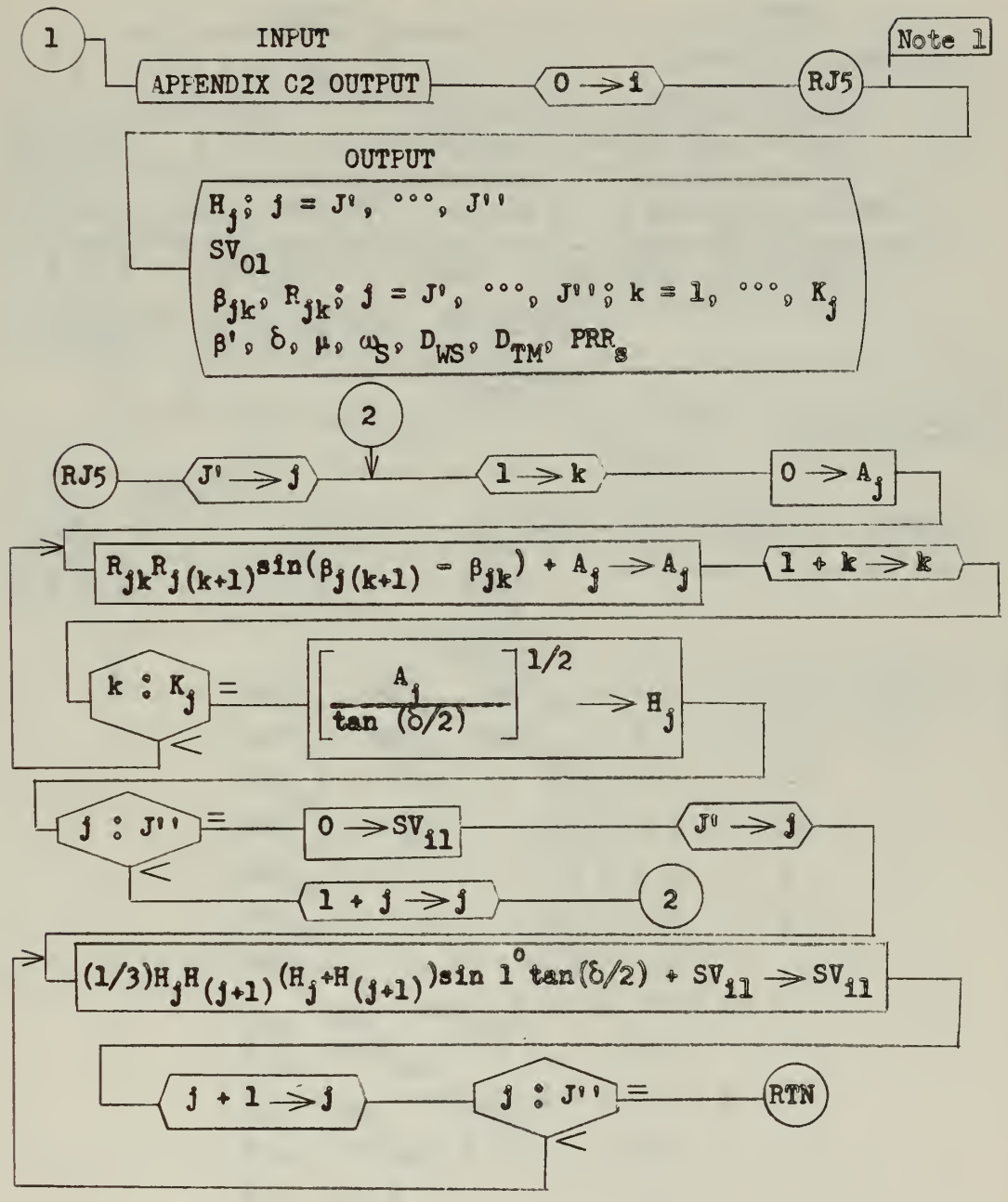
If to the right of the radial of maximum range K_j then $m = 2$ and the following angle will have a lesser k. There will be one or two radials of maximum range. That is

$$\beta_{j1K_j} \leq \beta_{j2K_j}$$

Note 13: If $\beta_{j1K_j} = \beta_{j2K_j}$, the program skips β_{j2K_j}. Otherwise the program includes it.

APPENDIX C3

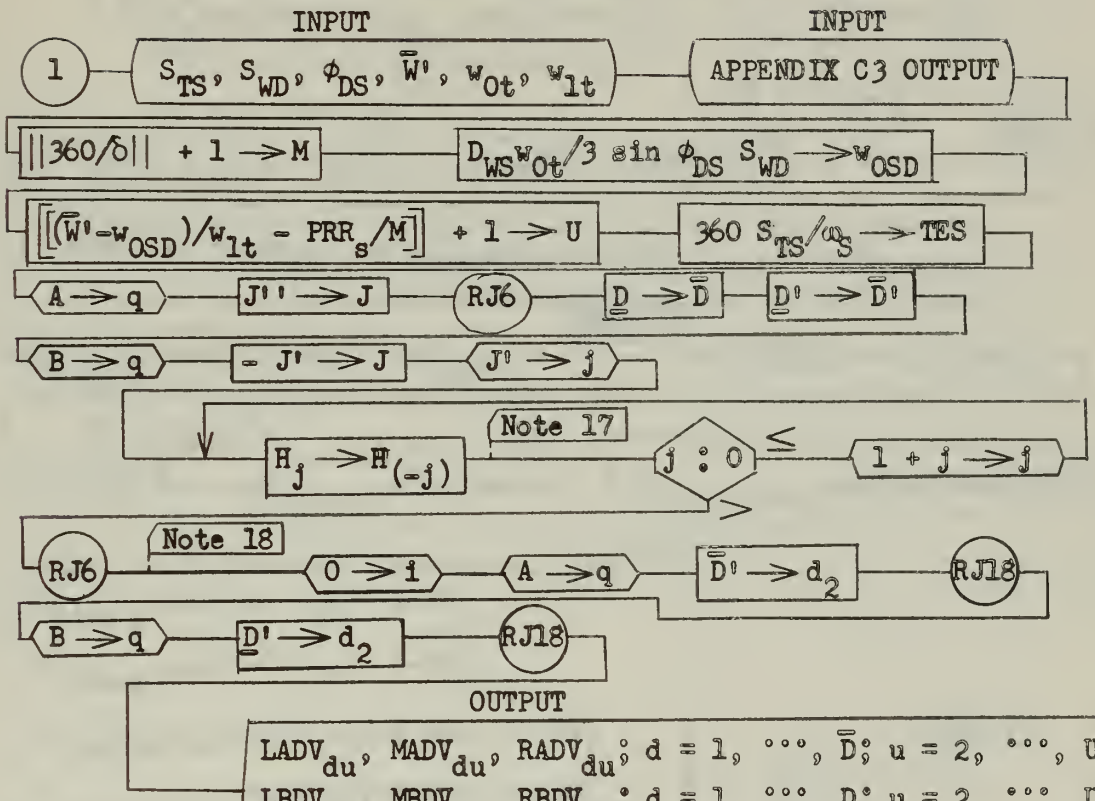
SCANNED VOLUMES; GENERAL CASE



Note 1: SV_{01} is not required in the program but it may be desired to print it here as information.

APPENDIX C4

SPECIAL VOLUME ELEMENTS: GENERAL CASE



OUTPUT

$LADV_{du}$, $MADV_{du}$, $RADV_{du}$; d = 1, ···, $\bar{D}$; u = 2, ···, U

$LBDV_{du}$, $MBDV_{du}$, $RBDV_{du}$; d = 1, ···, $\underline{D}$; u = 2, ···, U

AMV_{du}; d = $\bar{D}$+1, ···, $\bar{D}'$; u = 1, ···, U

BMV_{du}; d = $\underline{D}$+1, ···, $\underline{D}'$; u = 1, ···, U

ASV_{Odu}; d = 1, ···, $\bar{D}'$; u = 2, ···, U

BSV_{Odu}; d = 1, ···, $\underline{D}'$; u = 2, ···, U

qSV_{Ou}; q = A, B; u = 2, ···, U

h_{Ad}, B_{AId}, $\tan\phi_{AId}$; d = 1, ···, $\bar{D}'$

B_{AOd}, $\tan\phi_{AOd}$; d = 1, ···, $\bar{D}$

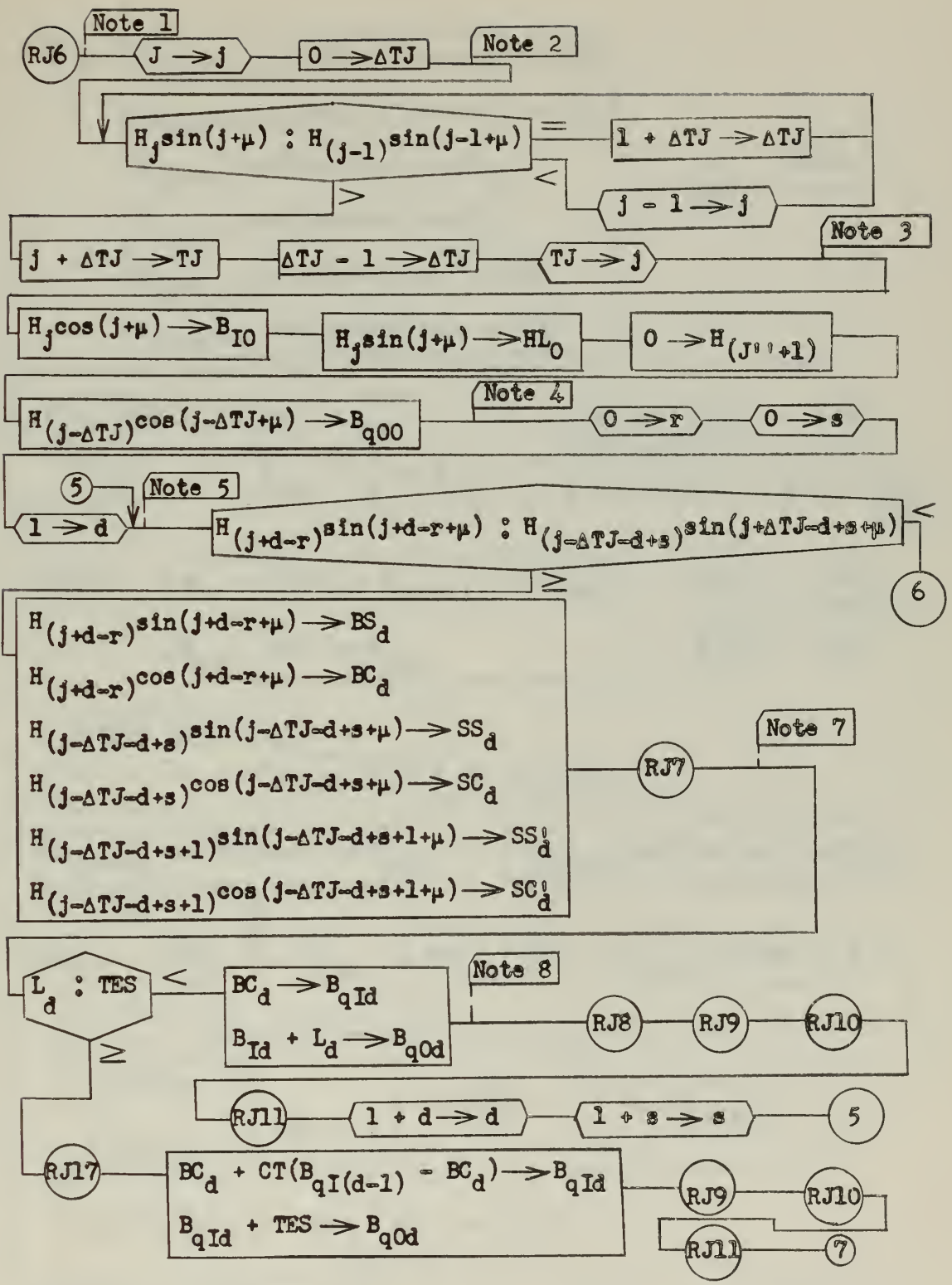
h_{Bd}, B_{BId}, $\tan\phi_{BId}$; d = 1, ···, $\underline{D}'$

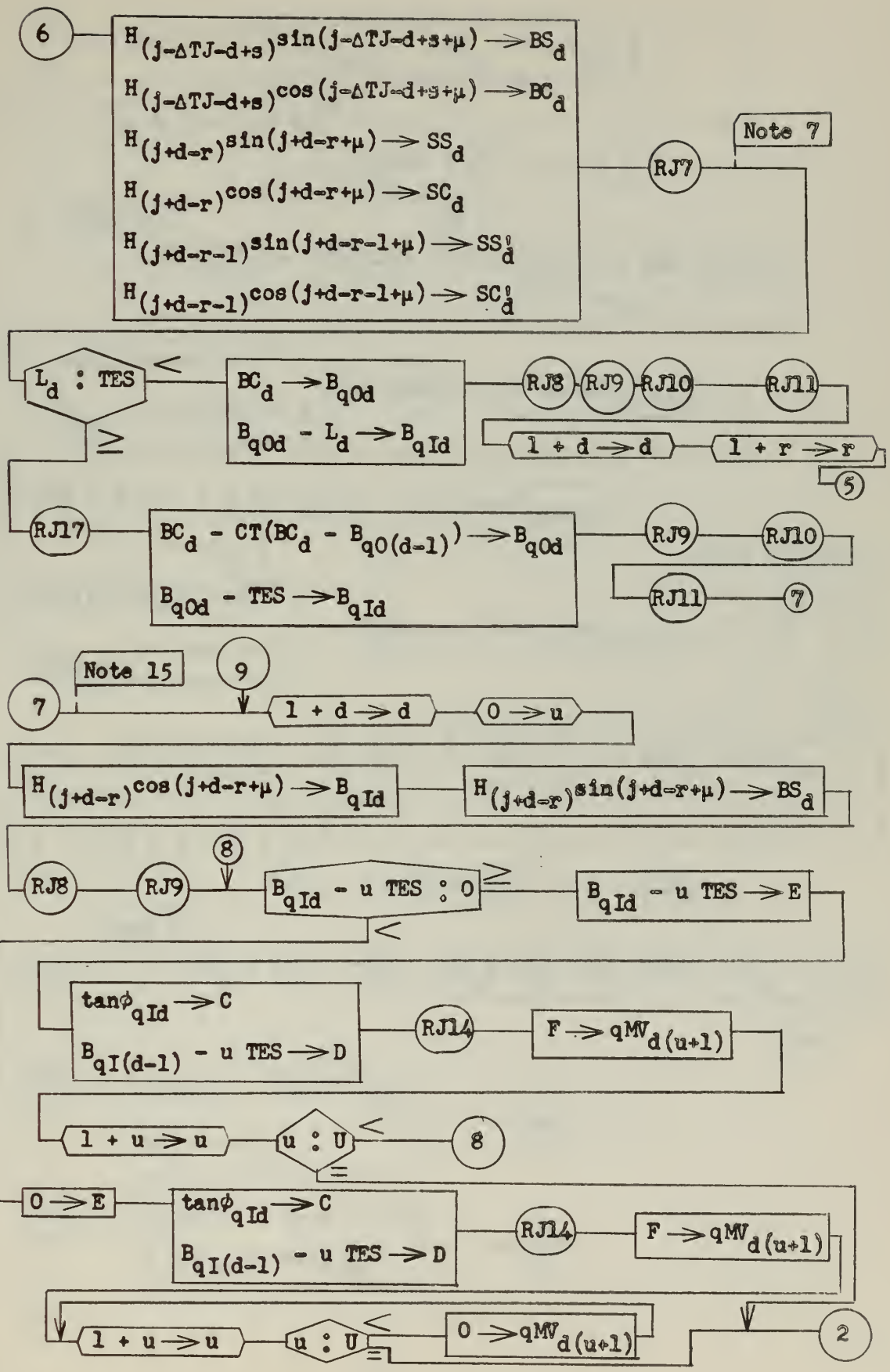
B_{BOd}, $\tan\phi_{BOd}$; d = 1, ···, $\underline{D}$

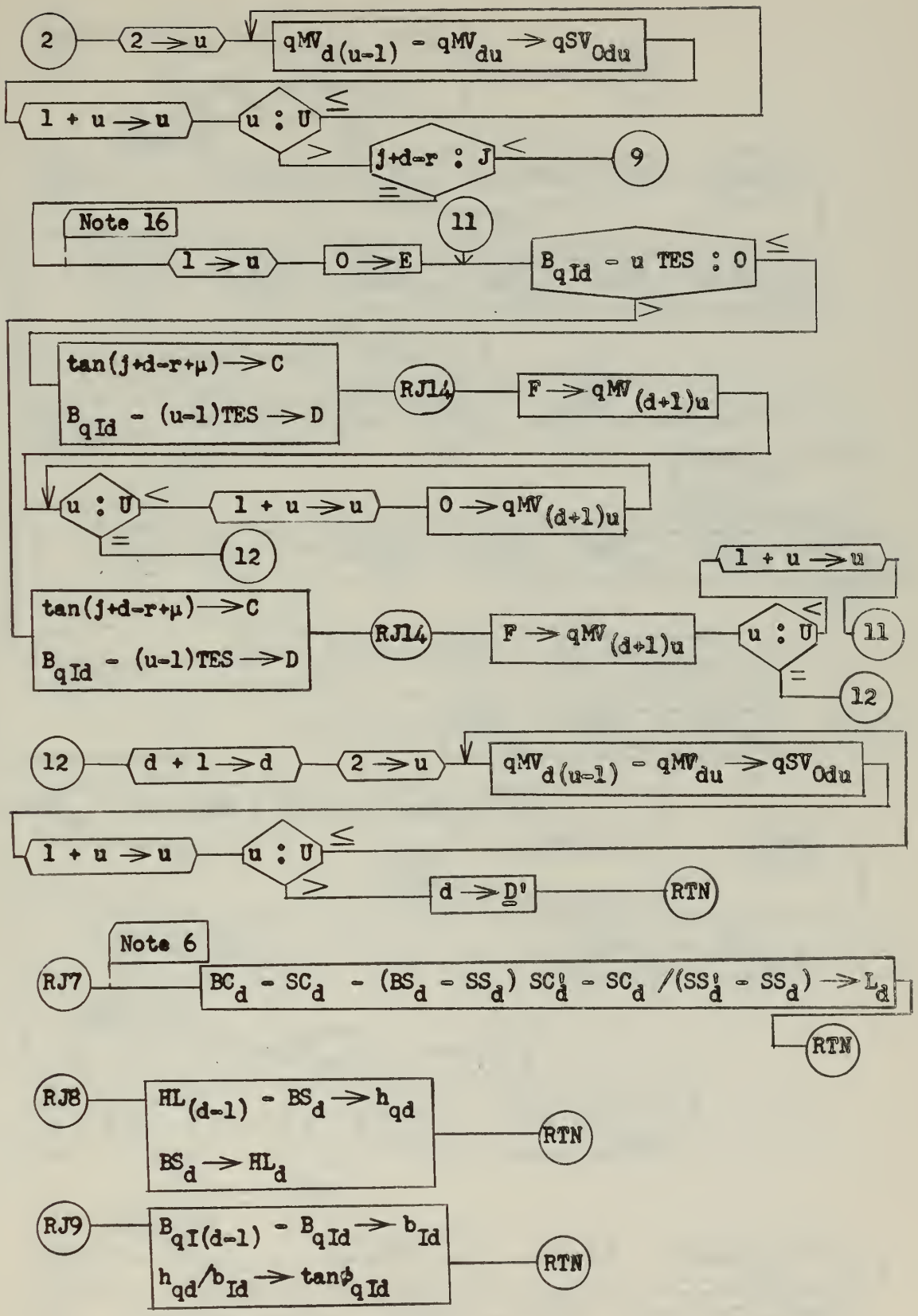
β_{jk}, R_{jk}; j = J', ···, J''; k = 1, ···, K_j

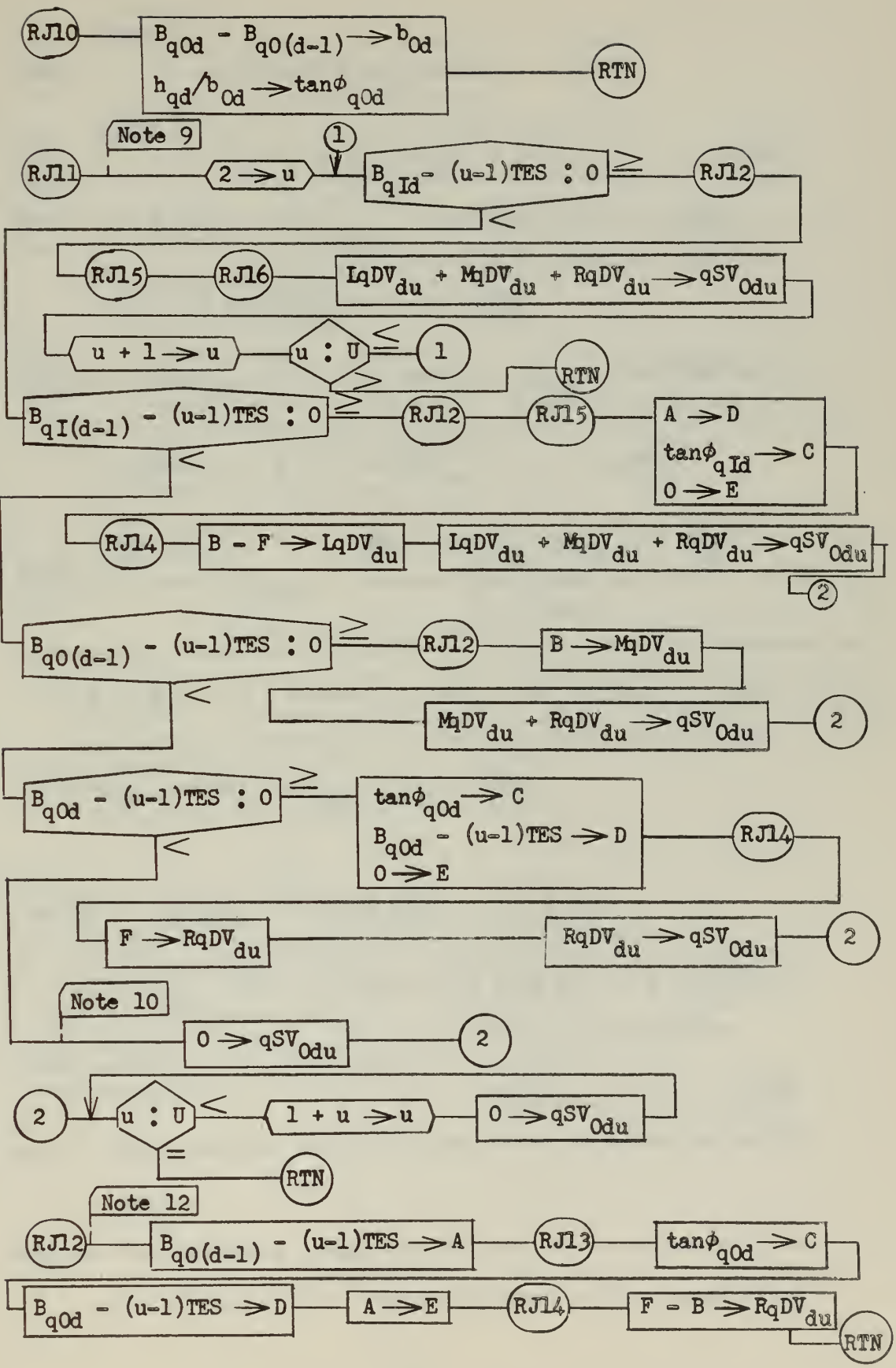
ϕ_{DS}, S_{WD}, S_{TS}, w_{OSD}, w_{1t}, M, U

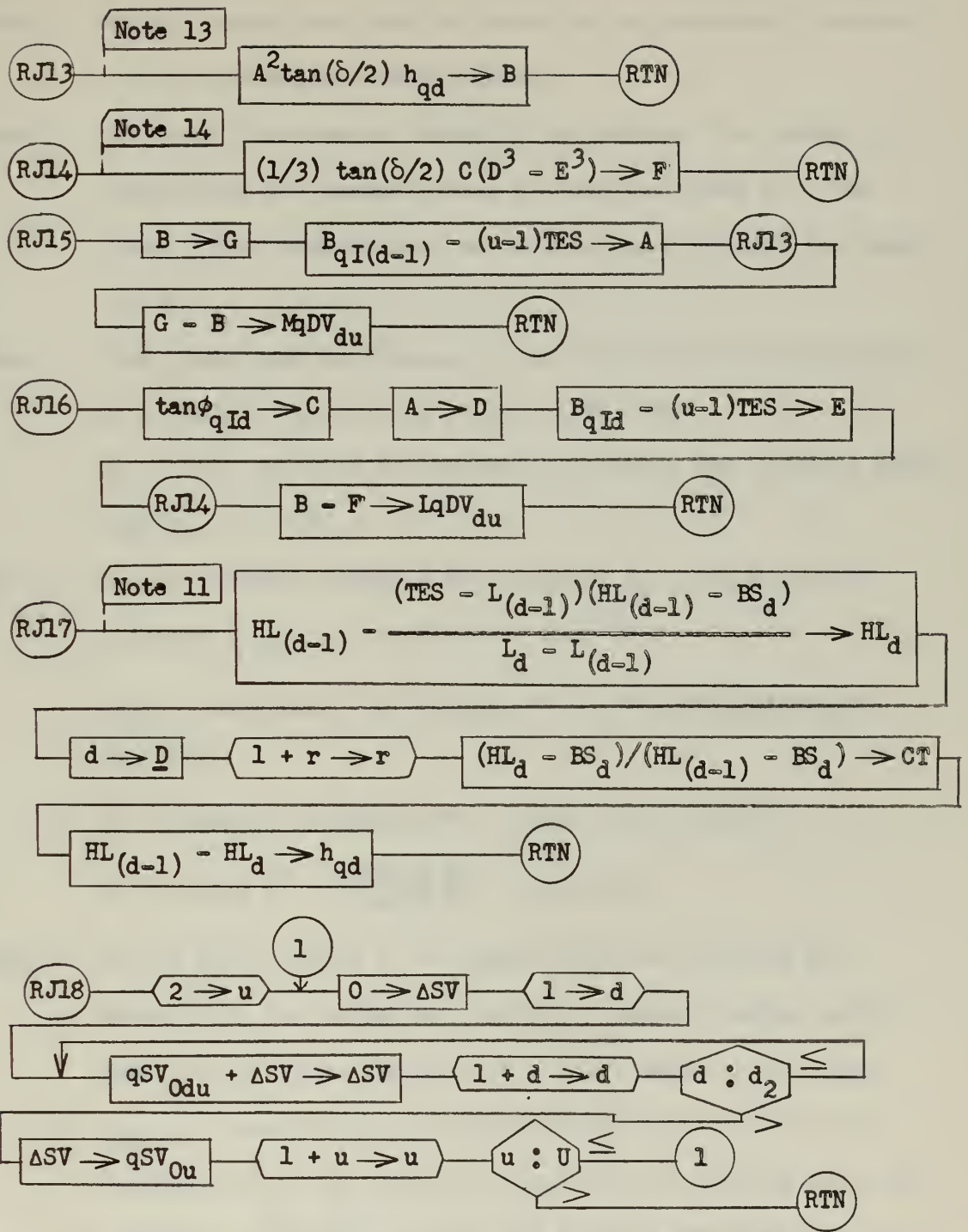
β', δ, μ, D_{WS}, D_{TM}, PRR_s


RJ6
Note 1
J → j
0 → ΔTJ
Note 2
$H_j \sin(j+\mu) : H_{(j-1)} \sin(j-1+\mu)$
=
1 + ΔTJ → ΔTJ
<
j - 1 → j
>
j + ΔTJ → TJ
ΔTJ - 1 → ΔTJ
TJ → j
Note 3
$H_j \cos(j+\mu) \rightarrow B_{I0}$
$H_j \sin(j+\mu) \rightarrow HL_0$
$0 \rightarrow H_{(J''+1)}$
$H_{(j-\Delta TJ)} \cos(j-\Delta TJ+\mu) \rightarrow B_{q00}$
Note 4
0 → r
0 → s
5
Note 5
1 → d
$H_{(j+d-r)} \sin(j+d-r+\mu) : H_{(j-\Delta TJ-d+s)} \sin(j+\Delta TJ-d+s+\mu)$
<
≥
6
$H_{(j+d-r)} \sin(j+d-r+\mu) \rightarrow BS_d$
$H_{(j+d-r)} \cos(j+d-r+\mu) \rightarrow BC_d$
$H_{(j-\Delta TJ-d+s)} \sin(j-\Delta TJ-d+s+\mu) \rightarrow SS_d$
$H_{(j-\Delta TJ-d+s)} \cos(j-\Delta TJ-d+s+\mu) \rightarrow SC_d$
$H_{(j-\Delta TJ-d+s+1)} \sin(j-\Delta TJ-d+s+1+\mu) \rightarrow SS'_d$
$H_{(j-\Delta TJ-d+s+1)} \cos(j-\Delta TJ-d+s+1+\mu) \rightarrow SC'_d$
RJ7
Note 7
$L_d : TES$
<
≥
$BC_d \rightarrow B_{qId}$
$B_{Id} + L_d \rightarrow B_{q0d}$
Note 8
RJ8
RJ9
RJ10
RJ11
1 + d → d
1 + s → s
5
RJ17
$BC_d + CT(B_{qI(d-1)} - BC_d) \rightarrow B_{qId}$
$B_{qId} + TES \rightarrow B_{q0d}$
RJ9
RJ10
RJ11
7


6
$H_{(j-\Delta TJ-d+s)}\sin(j-\Delta TJ-d+s+\mu) \rightarrow BS_d$
$H_{(j-\Delta TJ-d+s)}\cos(j-\Delta TJ-d+s+\mu) \rightarrow BC_d$
$H_{(j+d-r)}\sin(j+d-r+\mu) \rightarrow SS_d$
$H_{(j+d-r)}\cos(j+d-r+\mu) \rightarrow SC_d$
$H_{(j+d-r-1)}\sin(j+d-r-1+\mu) \rightarrow SS'_d$
$H_{(j+d-r-1)}\cos(j+d-r-1+\mu) \rightarrow SC'_d$
RJ7
Note 7
L_d : TES
$BC_d \rightarrow B_{qOd}$
$B_{qOd} - L_d \rightarrow B_{qId}$
RJ8
RJ9
RJ10
RJ11
1 + d → d
1 + r → r
5
RJ17
$BC_d - CT(BC_d - B_{qO(d-1)}) \rightarrow B_{qOd}$
$B_{qOd} - TES \rightarrow B_{qId}$
RJ9
RJ10
RJ11
7
Note 15
7
9
1 + d → d
0 → u
$H_{(j+d-r)}\cos(j+d-r+\mu) \rightarrow B_{qId}$
$H_{(j+d-r)}\sin(j+d-r+\mu) \rightarrow BS_d$
RJ8
RJ9
8
B_{qId} - u TES : 0
B_{qId} - u TES → E
$\tan\phi_{qId} \rightarrow C$
$B_{qI(d-1)}$ - u TES → D
RJ14
$F \rightarrow qMV_{d(u+1)}$
1 + u → u
u : U
8
0 → E
$\tan\phi_{qId} \rightarrow C$
$B_{qI(d-1)}$ - u TES → D
RJ14
$F \rightarrow qMV_{d(u+1)}$
1 + u → u
u : U
$0 \rightarrow qMV_{d(u+1)}$
2


2
2 → u
qMV d(u-1) - qMV du → qSV Odu
1 + u → u
u : U
≤
>
j+d-r : J
<
9
=
Note 16
11
1 → u
0 → E
B qId - u TES : 0
≤
>
tan(j+d-r+μ) → C
B qId - (u-1)TES → D
RJ14
F → qMV (d+1)u
u : U
<
1 + u → u
0 → qMV (d+1)u
=
12
1 + u → u
tan(j+d-r+μ) → C
B qId - (u-1)TES → D
RJ14
F → qMV (d+1)u
u : U
<
11
=
12
12
d + 1 → d
2 → u
qMV d(u-1) - qMV du → qSV Odu
1 + u → u
u : U
≤
>
d → D'
RTN
Note 6
RJ7
BC d - SC d - (BS d - SS d) SC' d - SC d /(SS' d - SS d) → L d
RTN
RJ8
HL (d-1) - BS d → h qd
BS d → HL d
RTN
RJ9
B qI(d-1) - B qId → b Id
h qd / b Id → tanφ qId
RTN


RJ10
$B_{qOd} - B_{qO(d-1)} \rightarrow b_{Od}$
$h_{qd}/b_{Od} \rightarrow \tan\phi_{qOd}$
RTN
Note 9
RJ11
$2 \rightarrow u$
1
$B_{qId} - (u-1)TES : 0$
RJ12
RJ15
RJ16
$LqDV_{du} + MqDV_{du} + RqDV_{du} \rightarrow qSV_{Odu}$
$u + 1 \rightarrow u$
$u : U$
1
RTN
$B_{qI(d-1)} - (u-1)TES : 0$
RJ12
RJ15
$A \rightarrow D$
$\tan\phi_{qId} \rightarrow C$
$0 \rightarrow E$
RJ14
$B - F \rightarrow LqDV_{du}$
$LqDV_{du} + MqDV_{du} + RqDV_{du} \rightarrow qSV_{Odu}$
2
$B_{qO(d-1)} - (u-1)TES : 0$
RJ12
$B \rightarrow MqDV_{du}$
$MqDV_{du} + RqDV_{du} \rightarrow qSV_{Odu}$
2
$B_{qOd} - (u-1)TES : 0$
$\tan\phi_{qOd} \rightarrow C$
$B_{qOd} - (u-1)TES \rightarrow D$
$0 \rightarrow E$
RJ14
$F \rightarrow RqDV_{du}$
$RqDV_{du} \rightarrow qSV_{Odu}$
2
Note 10
$0 \rightarrow qSV_{Odu}$
2
2
$u : U$
$1 + u \rightarrow u$
$0 \rightarrow qSV_{Odu}$
RTN
Note 12
RJ12
$B_{qO(d-1)} - (u-1)TES \rightarrow A$
RJ13
$\tan\phi_{qOd} \rightarrow C$
$B_{qOd} - (u-1)TES \rightarrow D$
$A \rightarrow E$
RJ14
$F - B \rightarrow RqDV_{du}$
RTN



Note 1: RJ 6 computes volumes either above or below the search depth.

Note 2: The following comparison is used to determine the plane j to be used as a reference plane for the computations.

Note 3: The following four function boxes set up reference constants for the subsequent computations.

Note 4: The index d is used to identify the levels. The index (d-r) identifies the current plane j above the plane TJ. The index (d-s) indentifies the current plane j below the plane TJ- ΔTJ.

Note 5: This comparison determines if the level d is to be determined by a plane $j > TJ$ or by a plane $j < (TJ - \Delta TL)$.
It is only required for depths at or above the critical depth, that is for $d \leq \bar{D}$.

Note 6: This subroutine computes the distance L_d. See Figure 21.
As shown $H_{(TJ + 2)} \sin (TJ + 2 + \mu) >$
$H_{(TJ-2)} \sin (TJ-2 + \mu)$ and $\Delta TJ = 0$. The distance to be computed is

L_d = length AB = length FG = length FE- length GE

$$L_d = \text{length FE} - \frac{\text{length BG}}{\text{length HI}} (\text{length HE})$$

Note 7: If the level length L_d is longer than the distance TES traveled by the submarine during a complete weapon search turn, the critical depth is at a level above L_d but below $L_{(d-1)}$. Then the level defined by TES must be found and designated $L_d = L_{\bar{D}}$ and the original level dropped from consideration until later. If L is shorter than TES then it is above the critical depth and its parameters must be computed to find qSV_{odu}, the volume searched in the d th level on the u th search turn. (q = A,B).

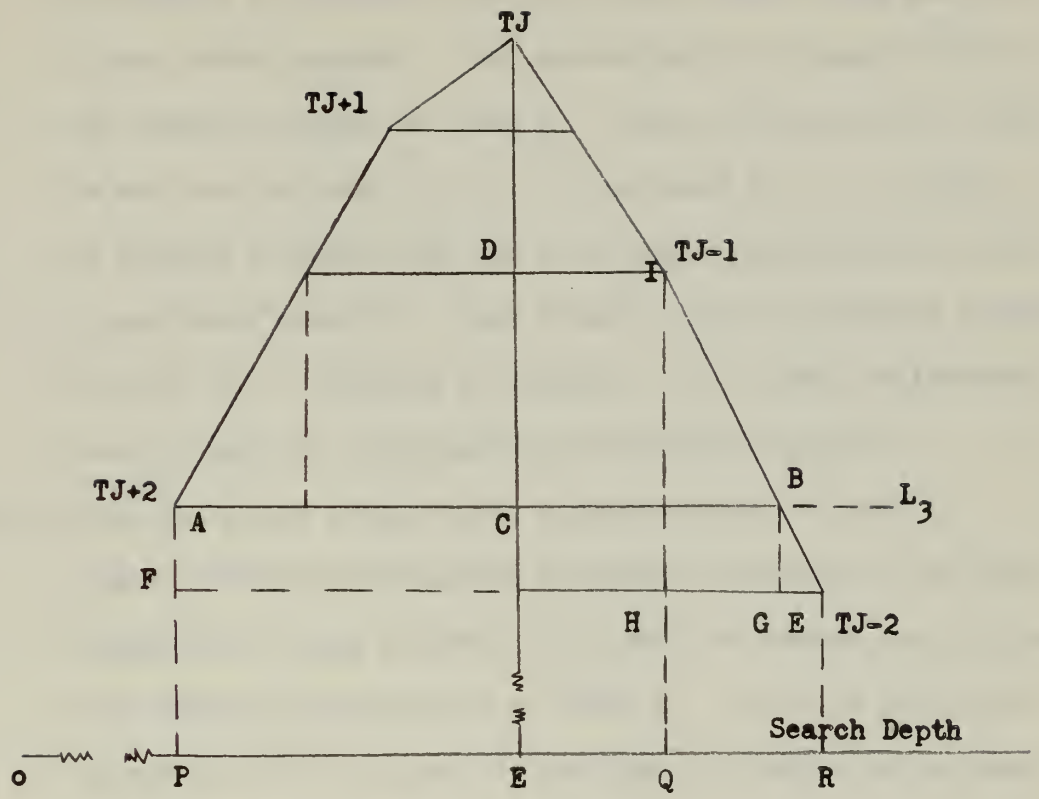


Figure 21: A sample volume portion for computation of L_d. The points $(TJ \pm d)$ are the radial endpoints of the corresponding planes. The level L_3 is under consideration and distance $(AB = L_3)$ is to be computed. The following are equivalent:

$BC_3 - SC_3$ = length FE

$BS_3 - SS_3$ = length BG

$SS'_3 - SS_3$ = length HI

$SC'_3 - SC_3$ = length HE

Note 8: See Figure 22. Subroutines RJ8, 9, and 10 compute the distances and angles on the right portion of the figure which have not already been computed. Then subroutine RJ11 computes all of the special volumes in level L_d. After RJ11 the level index as well as the index r or s, is increased by 1. If index s is changed it means that the plane which determined the level L_d was above plane TJ. Thus d and s must be increased together so that the difference is constant. It is their difference that is used for the comparison described by note 5.

Note 9: This subroutine computes the scanned volume in level L_d (where $d \leq \bar{D}$) on each search turn after the first. The first comparison is used to find out if the u th search turn is the last effective search turn in level L_d. If it is not, then subroutines RJ12, 15, and 16 are used to compute the volumes. The cross-section in this case looks like that of the right portion of Figure 22. If the u th search turn is the last of the effective search turns in level L_d, then the normal cross-section is cut at some point as in the left portion of Figure 22. In this case, the point where the cross-section is cut is found by one of the subsequent three comparisons and the calculation of scanned volume is modified accordingly.

Note 10: This corresponds to a vertical through the transducer which goes through a point in level L_d such as point S in Figure 22.

Note 11: This is used to compute the height of the critical depth above the search depth (length OR in Figure 22) and a correction factor to be used to find the constants for level $L_{\bar{D}}$.

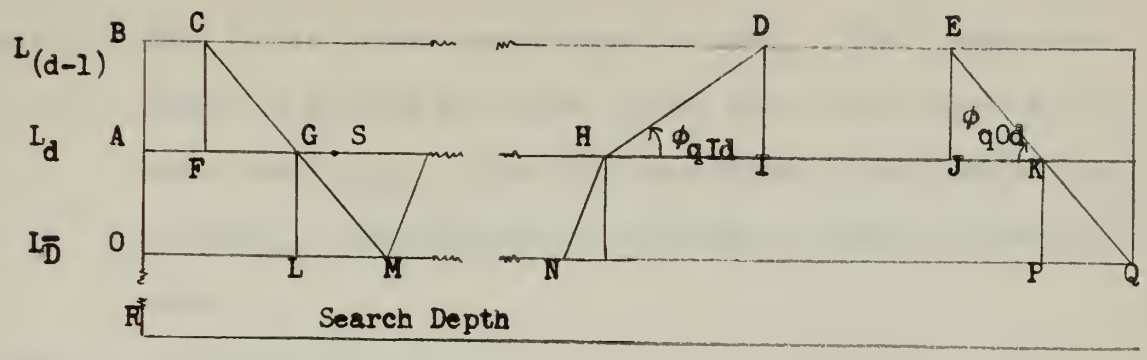


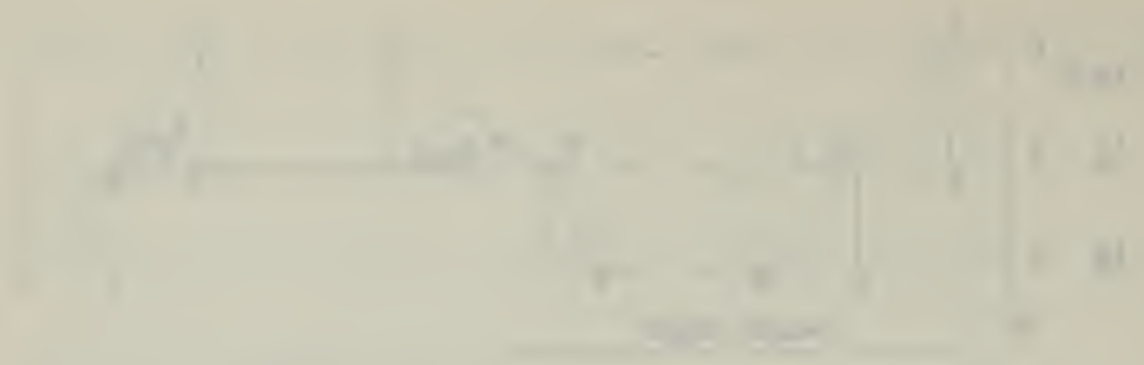
Figure 22: Vertical cross-section through levels $L_{\bar{D}}$ and L_d. The right portion depicts the volume scanned on the first search turn. The vertical OAB extends down to the transducer.

Volumes corresponding to depicted areas are

Area	Volume
HID	$LqDV_{dl}$
DEIJ	$MqDV_{dl}$
EJK	$RqDV_{dl}$
CFG = EJK	$RqDV_{du} \neq RqDV_{dl}$
ABCF $\neq$ DEIJ	$MqDV_{du} \neq MqDV_{dl}$
HDEK	qSV_{odl}
ABCG	qSV_{odu}

The distances used to compute volumes in level L_d are

length NQ = TES	length HI = b_{Id}
length BD = $B_{qI(d-1)}$	length JK = length FG = b_{od}
length BE = $B_{qo(d-1)}$	length AB = h_{qd}
length AH = B_{qId}	length AR = HL_d
length AK = B_{qod}	length BR = $HL_{(d-1)}$
length HK = L_d	length DE = $L_{(d-1)}$



Note 12: This is the volume computation for $RqDV_{du}$. This computation begins by turning the volume upside down so that level L_d is above level $L_{(d-1)}$. Then the computation is the same as that for qSV_{Odu} (corresponding to area ABDH in Figure 22) required later.

Note 13: See Figure 23.

Note 14: See Figure 24.

length A I = D $\tan\phi$

area IJK = $D^2 \tan(\delta/2)$

volume AJKI = (1/3) (area IJK) (length AI)

= $(1/3)\, D^3 \tan(\delta/2) \tan\phi$

Note 15: The volume computations (qMV_{du}) for levels below the critical depth are begun at this point.

Note 16: The volume computations for the last level use B_{qId} for the previous level and the angle j + d - r + μ because there is no plane j on which to base the calculations.

Note 17: The calculations below search depth are identical to those above after the H_j below are substituted for the H_j above.

Note 18: The final section computes the scanned volume on the u th search turn separately for above and below search depth.

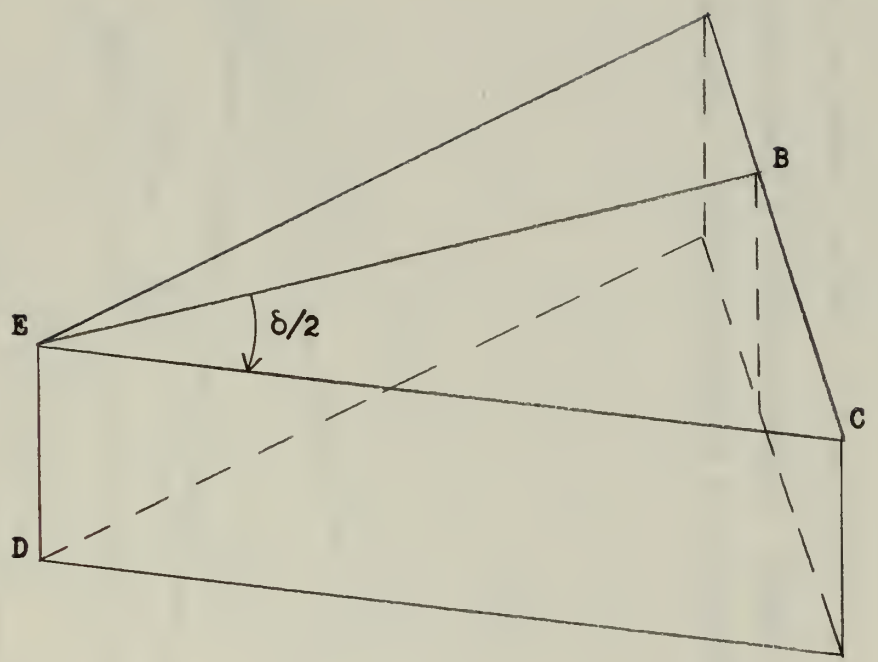


Figure 23: Volume computation of a triangular block. Length EB = A. Length BC = A tan (δ /2). Length ED = h_{qd}

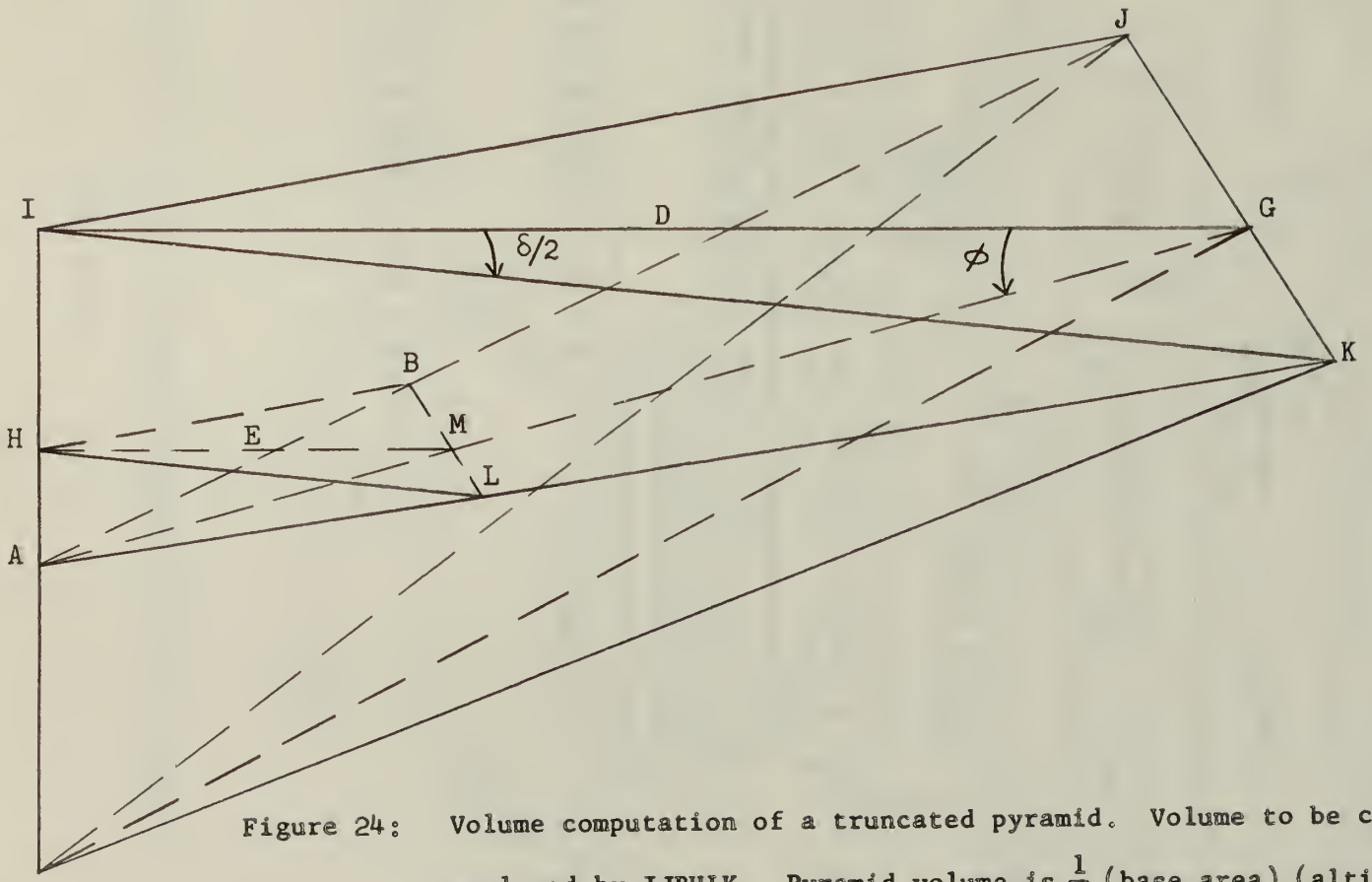
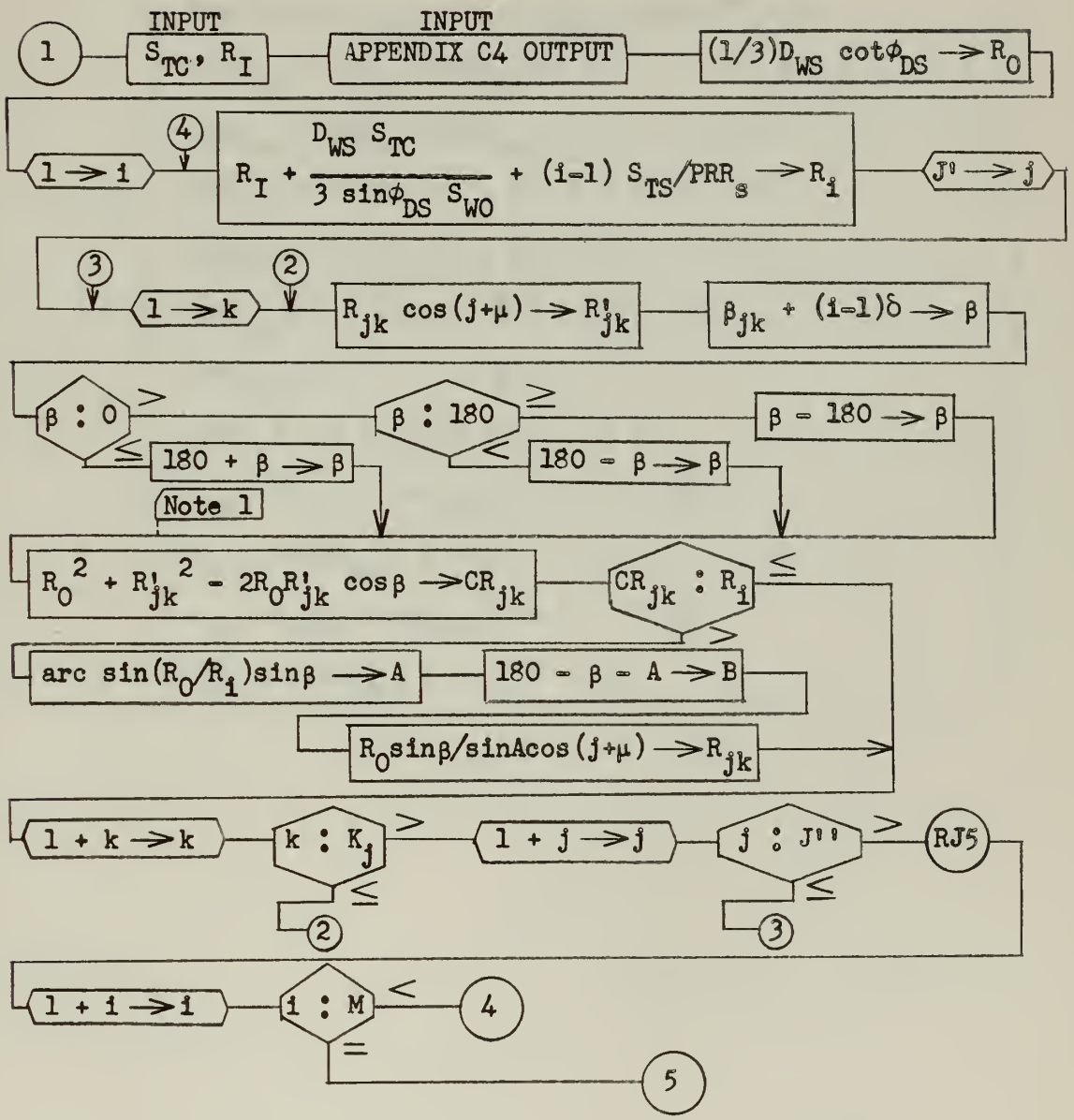


Figure 24: Volume computation of a truncated pyramid. Volume to be computed is enclosed by IJBHLK. Pyramid volume is $\frac{1}{3}$ (base area) (altitude). Length HM = E. Length IG = D. The base areas are IJK and HBL. The altitude lengths are AI and AH.

APPENDIX C5

FIRST TURN SCANNED VOLUME: i TH PING



Note 1: β is now the angle between R_o and the radial (R'_{jk}, β_{jk}).

5

OUTPUT

R_i, SV_{il}; i = 1, ···, M+1

R_O, M, U, δ, w_{OSD}, w_{lt}, D_{WS}, D_{TM}, PRR_s, β'

$LADV_{du}$, $MADV_{du}$, $RADV_{du}$; d = 1, ···, $\bar{D}$; u = 2, ···, U

$LBDV_{du}$, $MBDV_{du}$, $RBDV_{du}$; d = 1, ···, $\underline{D}$; u = 2, ···, U

AMV_{du}; d = $\bar{D}$+1, ···, $\bar{D}'$; u = 1, ···, U

BMV_{du}; d = $\underline{D}$+1, ···, $\underline{D}'$; u = 1, ···, U

ASV_{Odu}; d = 1, ···, $\bar{D}'$; u = 2, ···, U

BSV_{Odu}; d = 1, ···, $\underline{D}'$; u = 2, ···, U

qSV_{Ou}; q = A, B; u = 2, ···, U

h_{Ad}, B_{AId}, $\tan\phi_{AId}$; d = 1, ···, $\bar{D}'$

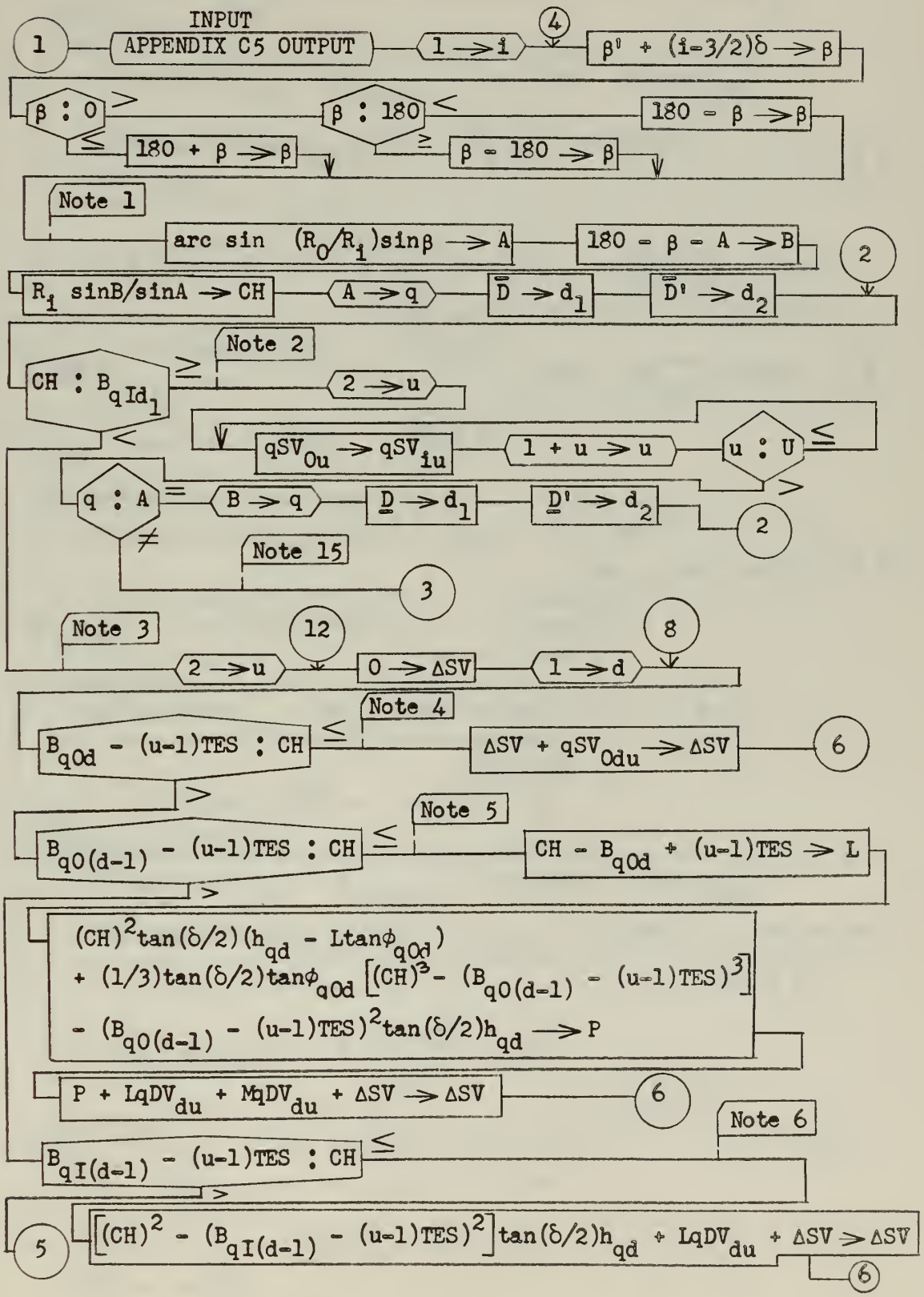
B_{AOd}, $\tan\phi_{AOd}$; d = 1, ···, $\bar{D}$

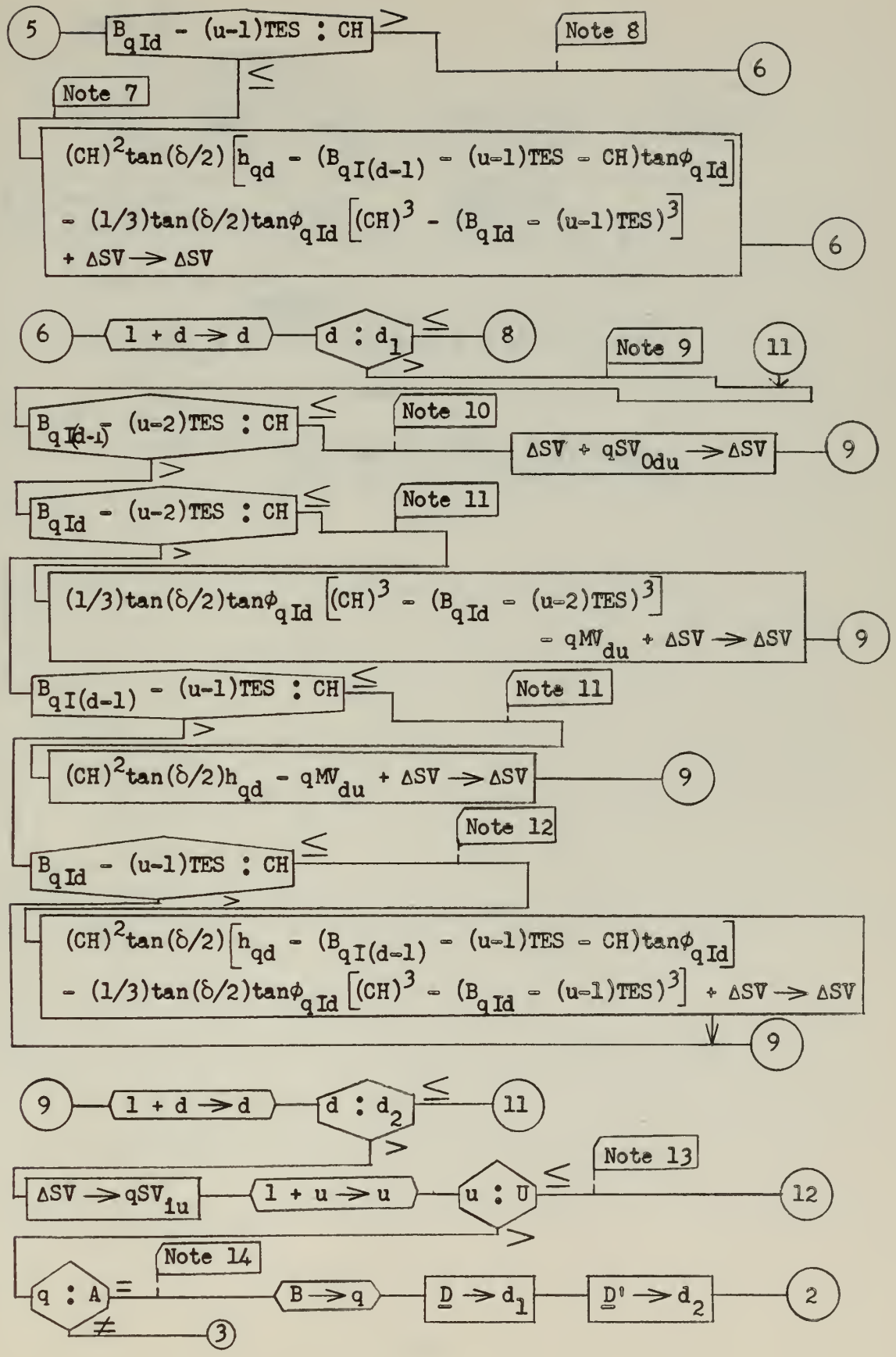
h_{Bd}, B_{BId}, $\tan\phi_{BId}$; d = 1, ···, $\underline{D}'$

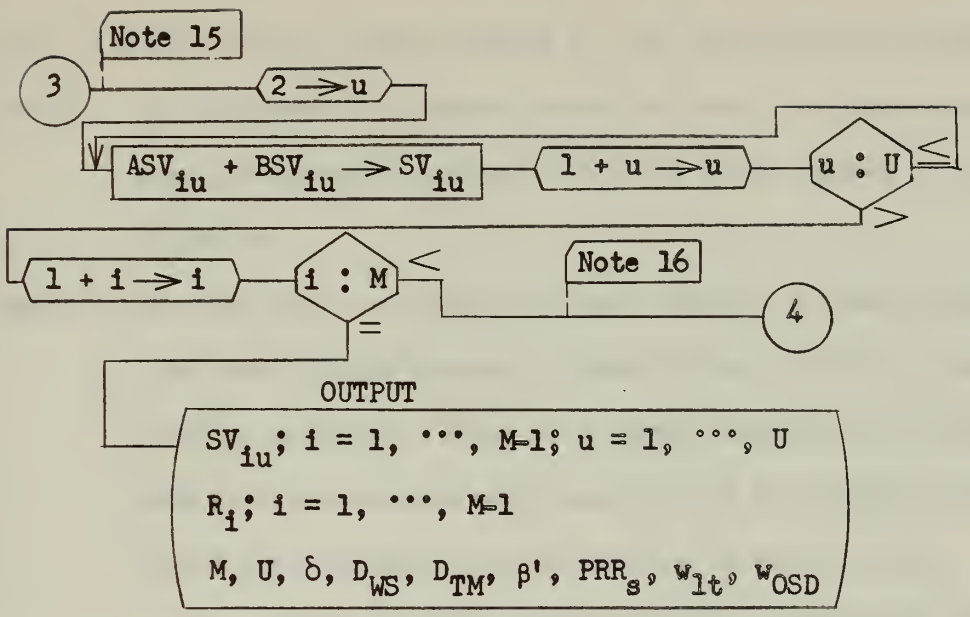
B_{BOd}, $\tan\phi_{BOd}$; d = 1, ···, $\underline{D}$

APPENDIX C6

SCANNED VOLUMES SUBSEQUENT TO FIRST TURN; i TH PING




5
B_{qId} - (u-1)TES : CH
>
Note 8
6
≤
Note 7
$(CH)^2\tan(\delta/2)\left[h_{qd} - (B_{qI(d-1)} - (u-1)TES - CH)\tan\phi_{qId}\right]$
$- (1/3)\tan(\delta/2)\tan\phi_{qId}\left[(CH)^3 - (B_{qId} - (u-1)TES)^3\right]$
$+ \Delta SV \rightarrow \Delta SV$
6
6
1 + d → d
d : d_1
≤
8
>
Note 9
11
$B_{qI(d-1)}$ - (u-2)TES : CH
≤
Note 10
$\Delta SV + qSV_{0du} \rightarrow \Delta SV$
9
>
B_{qId} - (u-2)TES : CH
≤
Note 11
>
$(1/3)\tan(\delta/2)\tan\phi_{qId}\left[(CH)^3 - (B_{qId} - (u-2)TES)^3\right]$
$- qMV_{du} + \Delta SV \rightarrow \Delta SV$
9
$B_{qI(d-1)}$ - (u-1)TES : CH
≤
Note 11
>
$(CH)^2\tan(\delta/2)h_{qd} - qMV_{du} + \Delta SV \rightarrow \Delta SV$
9
Note 12
B_{qId} - (u-1)TES : CH
≤
>
$(CH)^2\tan(\delta/2)\left[h_{qd} - (B_{qI(d-1)} - (u-1)TES - CH)\tan\phi_{qId}\right]$
$- (1/3)\tan(\delta/2)\tan\phi_{qId}\left[(CH)^3 - (B_{qId} - (u-1)TES)^3\right] + \Delta SV \rightarrow \Delta SV$
9
9
1 + d → d
d : d_2
≤
11
>
Note 13
$\Delta SV \rightarrow qSV_{iu}$
1 + u → u
u : U
≤
12
>
Note 14
q : A
=
B → q
$\underline{D} \rightarrow d_1$
$\underline{D}' \rightarrow d_2$
2
≠
3


Note 15
3
2 → u
ASV_iu + BSV_iu → SV_iu
1 + u → u
u : U
≤
>
1 + i → i
i : M
<
=
Note 16
4
OUTPUT
SV_iu; i = 1, ···, M-1; u = 1, ···, U
R_i; i = 1, ···, M-1
M, U, δ, D_WS, D_TM, β', PRR_s, w_lt, w_OSD

Note 1: β is now the angle between R_o and the bisector of the ith ping.

Note 2: In this case the target volume cylinder is large enough such that the scanned volumes after the first search turn are not effected.

Note 3: In this case the target volume cylinder is small enough such that some of the scanned volumes after the first search turn must be modified. This is accomplished above and below the search depth and for each search turn by summing the modified or unmodified volumes over the levels to obtain qSV_{iu}. First each search turn above search depth is considered, then each search turn below.

Note 4: In this case the original qSV_{odu} is not changed.

Note 5: In this case the original $LqDV_{du}$ and $MqDV_{du}$ are not changed but $RqDV_{du}$ is modified and must be recalculated. The original cross-section appears in Figure 25. Using formulas of subroutines RJ13 and RJ14 the new volume P is P = volume FGED + volume ADEC - volume ABGF.

Note 6: In this case the original $RqDV_{du}$ is zero, the original $LqDV_{du}$ is unchanged and the modified $MqDV_{du}$ is computed from the formula for subroutine RJ13.

Note 7: In this case the original $MqDV_{du}$ and $RqDV_{du}$ are zero and the modified $LqDV_{du}$ is computed from the formulas for subroutines RJ13 and 14 as the difference of two volumes.

Note 8: In this case the contribution of level L_d on the u th search turn is zero.

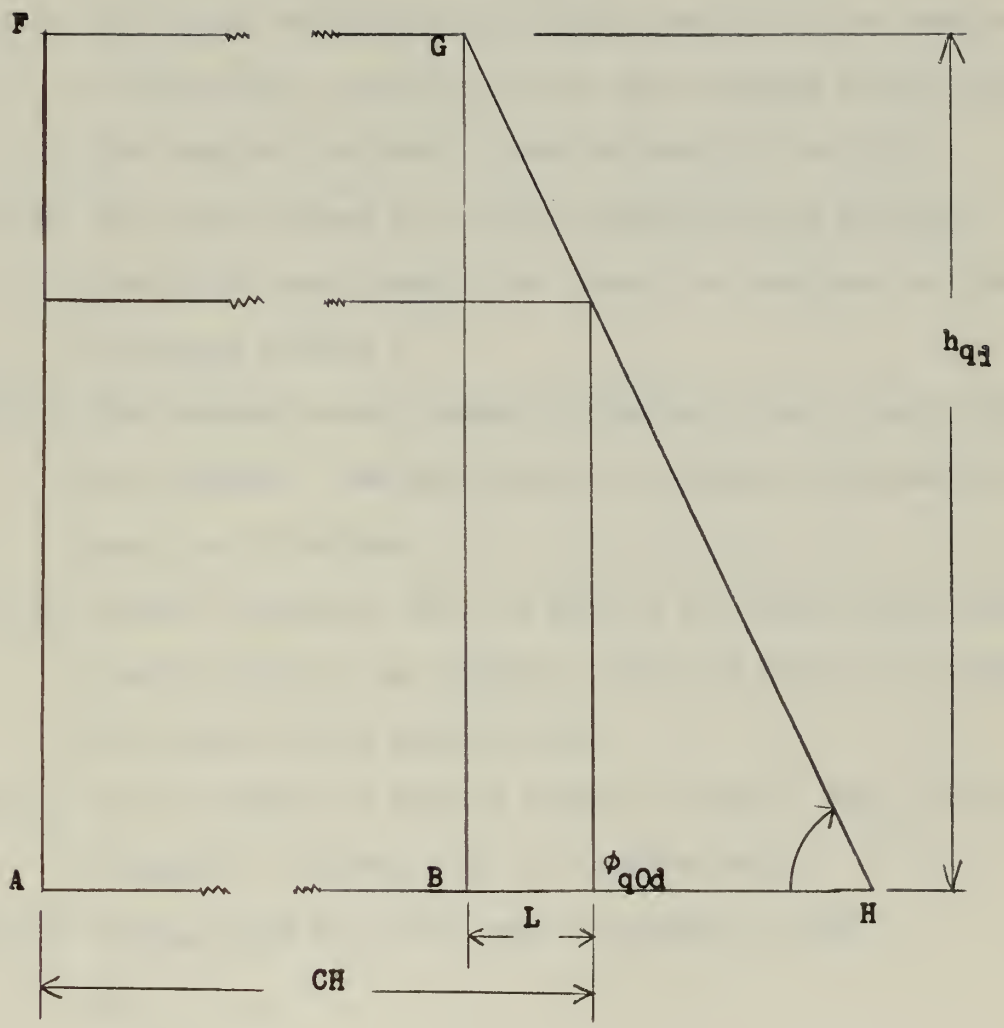


Figure 25: Computation of modified volume $RqDV_{du}$. Length FG is $B_{qO(d-1)} - (u - 1)$ TES. Area BGH represents the original volume $RqDV_{du}$. Line AF is part of the vertical through the weapon.

Note 9: The contributions of levels above the critical depth have now been determined.

Note 10: The contribution of level L_d is not changed.

Note 11: The missed volume on the previous search turn has been reduced leaving less to be picked up on the unchanged second turn. The formulas are those of subroutines RJ13 and RJ14.

Note 12: The missed volume in both the current and the previous search has been reduced. The formula is identical to that discussed in Note 7.

Note 13: The scanned volumes indexed by one set of q, u, and i are now computed. The next step is to change u and compute the next set of volumes.

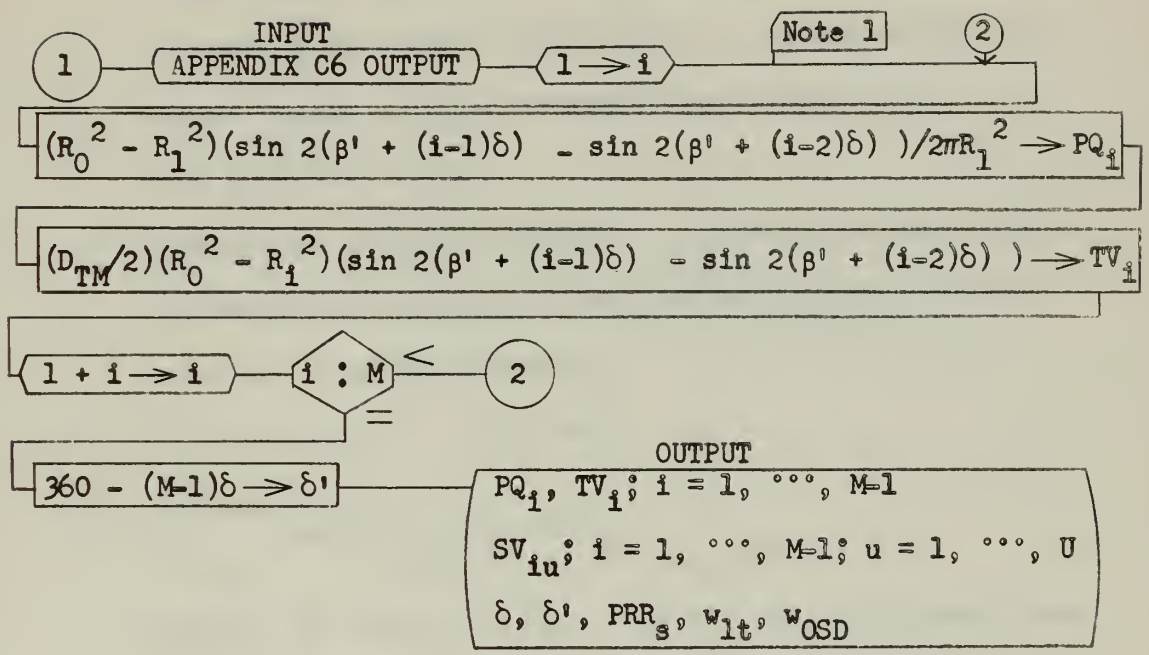
Note 14: Scanned volumes on the i th ping of all search turns above search depth are now computed. The next step is to compute the volumes below search depth.

Note 15: The increments of scanned volume for ping i, qSV_{iu}, are now computed. The next step is to compute SV_{iu}.

Note 16: Having found SV_{iu}, the index is changed to find $SV_{(i+1)u}$.

APPENDIX C7

TARGET VOLUME: i TH PING



Note 1: The problem is to compute areas as depicted horizontally and vertically lined in Figure 16. Using Figure 16, with origin at point C and x axis toward point C, the equation of the target volume cylinder circumference on ping i is

$$(x + R_o)^2 + y^2 = R_i^2$$

Changing to polar coordinates (r, β) with the same origin and measuring angles positive counterclockwise from the x axis, the equation becomes

$$r^2 + (2R_o \cos\beta) \cdot r - (R_i^2 - R_o^2) = 0$$

Solving for r,

$$r = R_i \sqrt{1 - (R_o/R_i)^2 \sin^2\beta} - R_o \cos\beta$$

To find the area for ping i, $r(r\, d\beta)$ is integrated from $\beta' + (i - 2)\delta$ to $\beta' + (i - 1)\delta$.

Thus

$$\text{area} = \int_{\beta'+(i-2)\delta}^{\beta'+(i-1)\delta} r^2\, d\beta$$

Let $\beta' + (i-1)\delta = B$

$\beta' + (i-2)\delta = A$

Then

$$\int_A^B r^2\, d\beta = \int_A^B \left[(R_i^2 - R_o^2) + 2R_o^2 \cos^2\beta - 2R_oR_i\sqrt{1 - (R_o/R_i)^2 \sin^2\beta}\cos\beta\right] d\beta$$

$$\int_A^B (R_i^2 - R_o^2)\, d\beta = (R_i^2 - R_o^2)(B - A)$$

$$\int_A^B 2 R_o^2 \cos^2\beta \, d\beta = R_o^2 \left[\beta + (1/2) \sin 2\beta\right]_A^B$$

Let $x = \sin\beta$

Then $dx = \cos\beta \, d\beta$

$$-2 R_o R_i \int_A^B \sqrt{1 - (R_o/R_i)^2 \sin^2\beta} \cos\beta \, d\beta$$

$$= -2 R_o R_i \int_A^B \sqrt{1 - (R_o/R_i)^2 x^2} \, dx$$

Let $\sin\beta = (R_o/R_i)\, x$

Then $\cos\beta \, d\beta = (R_o/R_i) \, dx$

$$-2 R_o R_i \int_A^B \sqrt{1 - (R_o/R_i)^2 x^2} \, dx = -2 R_i^2 \int_A^B \cos^2\beta \, d\beta$$

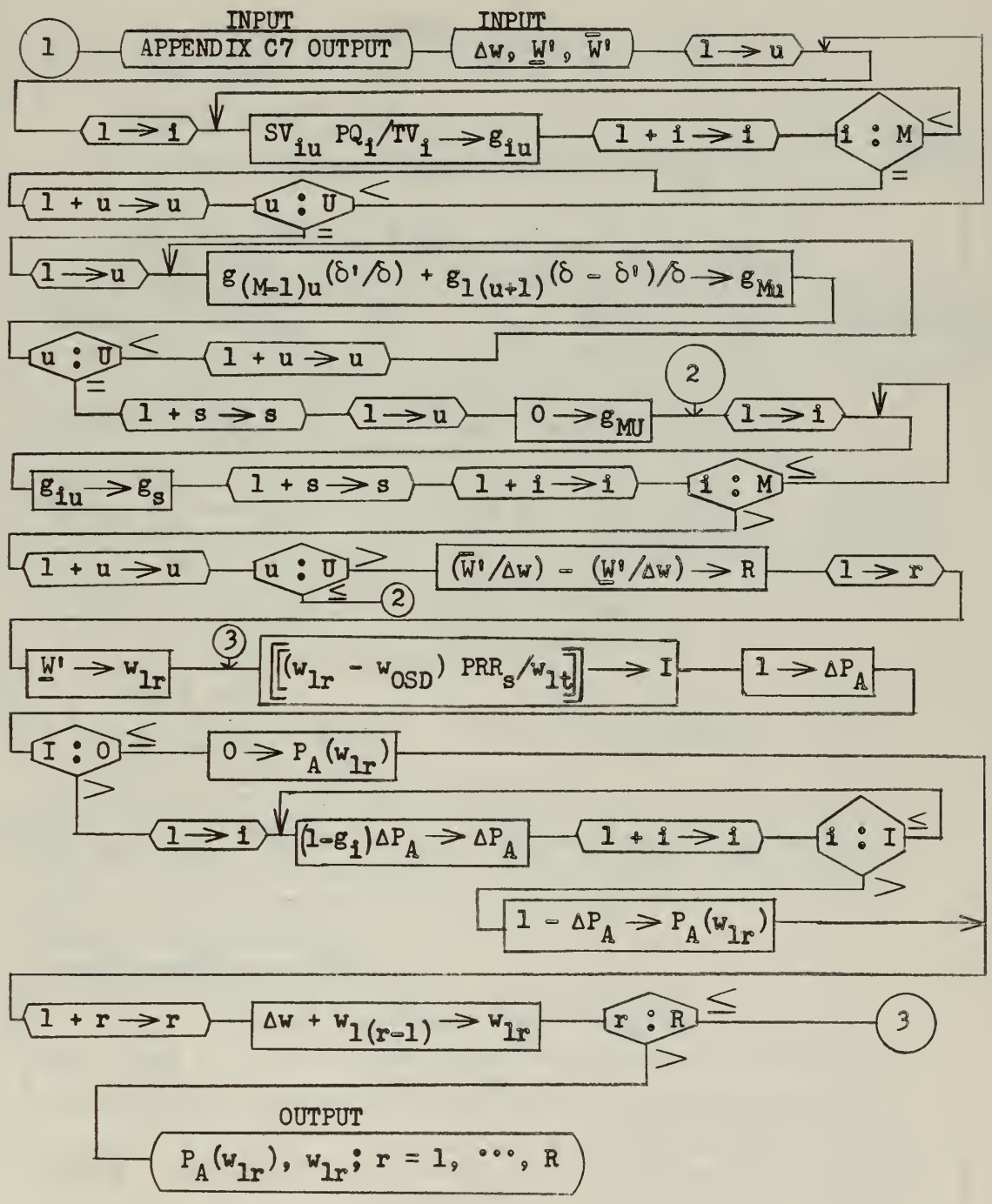
$$= -R_i^2 \left[\beta + (1/2)\sin 2\beta\right]_A^B$$

Therefore:

$$\int_A^B r^2 \, d\beta = (R_o^2 - R_i^2)\, (\sin 2B - \sin 2A) / 2$$

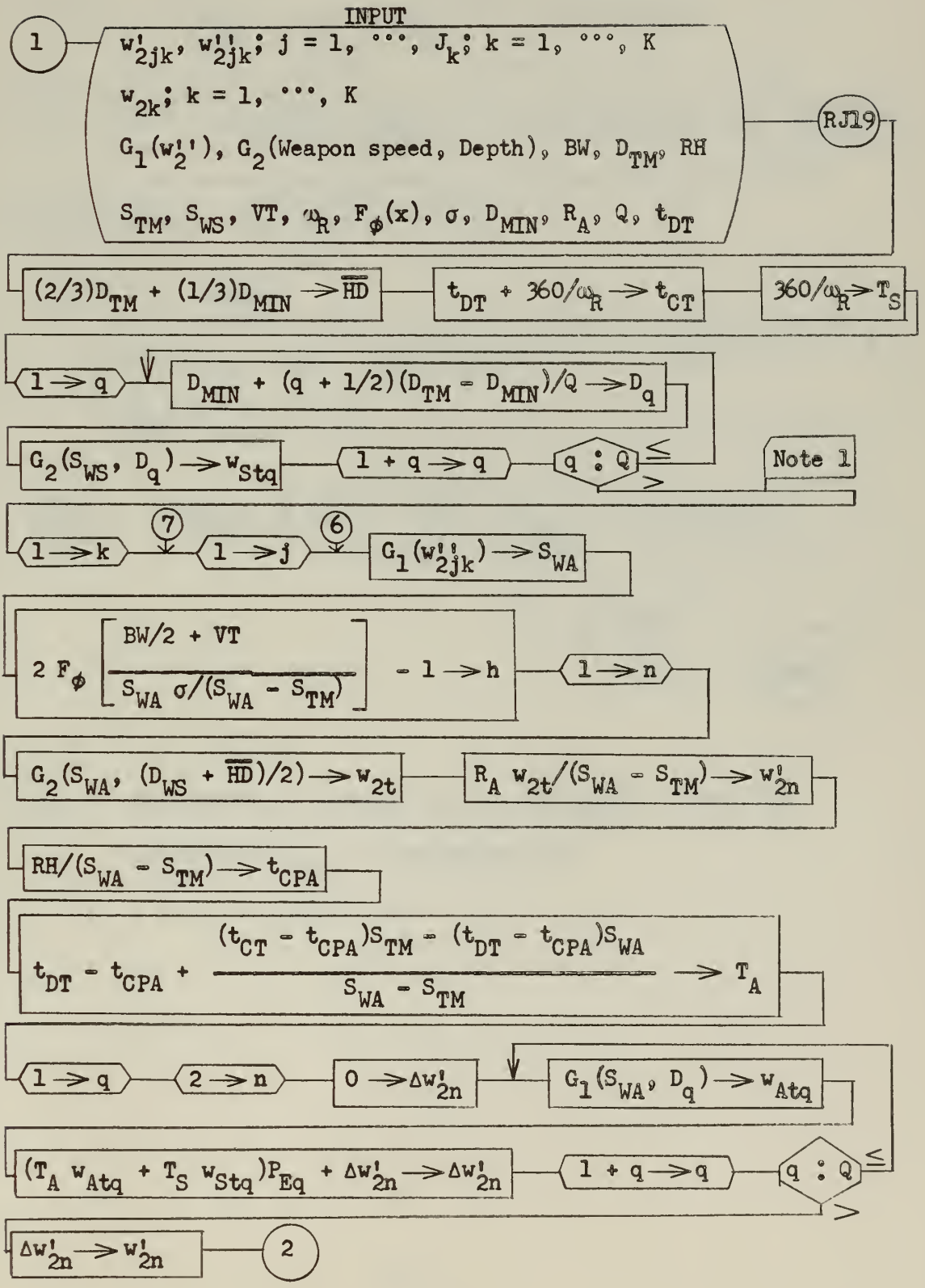
APPENDIX C8

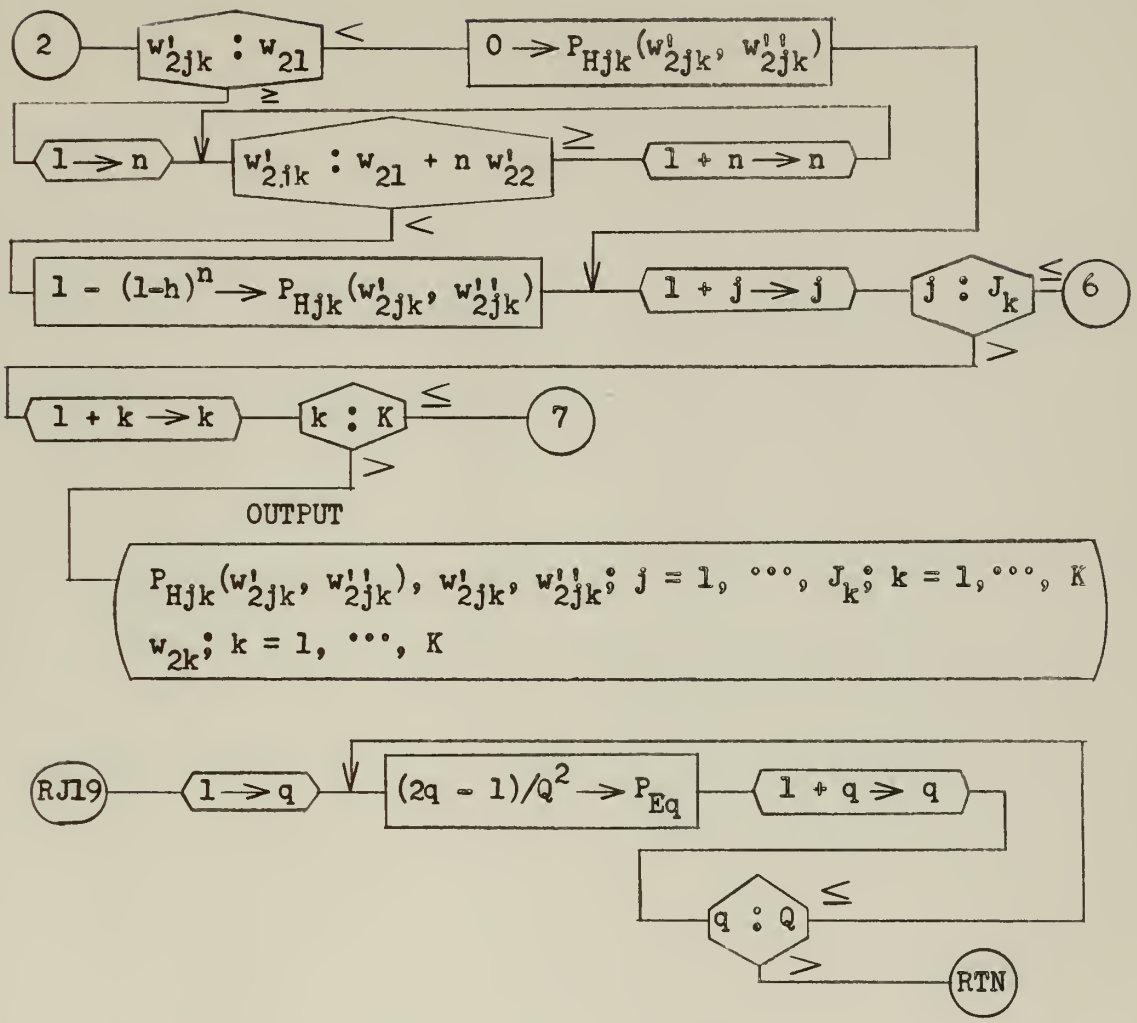
ACQUISITION PROBABILITIES: I PINGS



APPENDIX D

PROBABILITY OF HIT





Note 1: The problem constants are now determined. Next the probability P_{Hjk} is determined for $j = 1, \cdots, J_k$; $k = 1, \cdots, K$.

thesH192
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